Political Economy of Debt and Growth
Appendix

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In this Appendix we present additional results for our paper "Political Economy of Debt and Growth." In Section 1 we (1) offer a discussion on the relationship to Battaglini and Coate [2008] framework; (2) discuss the relevance of our results for open economies; and (3) elaborate on how the model can be adopted for empirical analysis at high and medium frequencies. In Section 2 we present derivations and proofs of the claims made in the paper. In Section 3 we describe our data sources for the calibration. In Section 4 we develop a version of the model with no growth and quasi-linear preferences. Section 5 offers additional information about the robustness of our numerical results.

1 A discussion

1.1 On the relationship with Battaglini and Coate [2008] framework

Our model finds its inspiration in the political economy model of debt by Battaglini and Coate [2008] (henceforth BC), but it departs from it in many significant ways. Apart from the fact that BC never considered growth, the main difference is in the way the policy space is modelled. While in BC politicians gain political power by distributing lump-sum transfers, in our model politicians distribute local public goods. It is important to understand why, and the extent to which, this change affects the results.

The first implication of considering local public goods is that they may generate externalities: a bridge constructed in county j will affect neighboring county i, even if the bridge was provided to j as a form of pork. While this expands the scope of the model, it does not change the essence of the political conflict, and, hence, it is not crucial for our analytical results. The presence of the externality is important from a quantitative point of view, as discussed in Section 4.2 of the paper: a reduction in α implies an increase in the externality and a decrease in the political conflict. Tables 2 and 4 of the paper show that differences in α translate into significant differences in political and economic outcomes.[1]

The feature of local public goods that changes the nature of the problem is the fact that they enter in the citizens' utilities in a non-linear way (transfers in BC enter the utility functions linearly). This small change has important implications for the analysis. To see why this is the case note first that as debt increases, the marginal cost of public funds increases. The marginal cost of public funds is a convex function of b and converges to infinity as debt approaches the upper bound $\overline{b}$. When the marginal utility of transfers is linear, this implies that above some (high) level of debt, there are no pork transfers. That is, the economy enters what BC call the *Responsible Policy Making* regime (RPM): the debt level does not rise even when the policy is decided by self-interested politicians.[2] When, as in our model, the marginal utility of transfers (public goods) is non-linear and satisfies the Inada conditions, the economy never reaches the RPM regime because the lower is the level of public goods the higher their marginal benefit is.[3] The only reason why debt does not converge to $\overline{b}$ is that the politicians want to keep interest rates low, a phenomenon that is absent in BC. In Section 4 of this Appendix we formalize this point by presenting a version of our model in which citizens' utility is linear in consumption, but non-linear in the local public goods. As in BC, this utility implies a constant interest rate, and hence there is no interest rate manipulation channel. As a result, the debt level is ever-growing, converging to its upper bound.

1.2 Closed versus open economies

In the previous analysis, we have focused on a closed economy. In this context, the government has the monopoly on government bonds and so changes in debt have the maximal effect on the interest rate. In an open economy the government competes for funds with other countries: it is reasonable to assume that the elasticity of the interest rate to debt would be lower in these cases. The analysis

[1] We stress that in our model the ultimate determinant of the political conflict is not α, but $\alpha\frac{\mathcal{G}}{n}Q(\mathcal{G},q)$. To this end, the externality can be disposed off by setting α to 1 and re-calibrating the remaining parameters, including q and $\mathcal{G}$ which are fixed in the benchmark calibration, to match the targeted moments in the data.

[2] In fact, in BC and in the closely related model of Barseghyan, Battaglini and Coate [2013] and Battaglini [2014], debt in RPM regime may decline due to the precautionary savings motive arising from random shocks.

[3] Technically, this statement requires the lower bound on public goods, $\underline{g}$, to be small. However, our upper bound on debt $\overline{b}$ is defined taking into account that at least $\underline{g}$ amount of public goods should be provided to all districts. Hence, the statement is true for all interior b.

in the paper suggests that in these environments the government has lower incentives to keep debt small. If the elasticity of the interest rate to debt is lower (but still positive), then we should expect a higher level of debt accumulation;[4] in the limit case in which by opening the economy to the world capital markets the elasticity becomes zero, we should expect debt to converge to $\overline{b}$. In the Section 4 of this Appendix we formalize this point by presenting a variation of the basic model in which interest rates are completely insensitive to domestic policies. As expected, public debt converges to $\overline{b}$.

We should however stress that for the existence of an interior balanced growth level of debt, Proposition 2 of the paper only requires positive elasticity at the steady state, not globally. Even if for low levels of debt the elasticity is zero (as it may be for some countries, at least at a given point in time), it could become positive when public debt grows. There is evidence that seems to confirm that the sensitivity of the interest rate to debt plays an important role in shaping fiscal policy even for an economy fully integrated into the world capital markets and small risk of default such as the U.S.[5]

1.3 Towards an empirical analysis at high and medium frequencies

In our paper we abstracted from shocks, and, hence, our model cannot be immediately used in standard empirical macroeconomic research. However, we view it as a fundamental "ready-to-use" block for applied DSGE models that allows for endogenous fiscal policy – something that, to our knowledge, has been missing from these models. Consider, for example, a standard approach of comparing/fitting impulse response functions generated by the model to those estimated in the data (e.g., Christiano, Eichenbaum and Evans [2005]). A crucial step in this literature is to translate a DSGE model into a system of linear difference equations describing the evolution of the variables of interest by linearizing the system of the equations implied by the model around the non-stochastic steady state. Importantly, our solution procedure – constructing global policy functions – automatically determines what the non-stochastic steady state is. Moreover, when exogenous stochastic state variables are i.i.d., the entire impulse response functions, up to a scale parameter, are described by the linearized policy functions, $p(b)$, except of course for the period of the initial shock.

In sum, our model offers an endogenous theory of fiscal policy that can be further enriched to allow for temporary shocks. It can be easily adopted to carry out empirical analysis at high and medium frequencies.

[4] Azzimonti, de Francisco, and Quadrini [2014] argue that the liberalization of financial markets of the past thirty years has contributed to the rise in public debt across OECD countries as well as the decline in public debt elasticity of the interest rate. They also provide evidence of a positive correlation between public debt and some indices of capital market liberalization.

[5] For evidence on this see the memories of former Treasury Secretary Robert Rubin (Rubin [2003]).

2 Derivations and proofs

2.1 Citizens' problem

For a given sequence of government policies, citizens' maximization problem in period 0 is:

$$\max_{\{c_t,S_t,l_t\}} \sum_{t=0}^{\infty} \delta^t \left\{ \log\left(C_t(1-l_t)^{\mu}\right) + \omega \log\left[\left(\gamma_t^i\right)^{\alpha} \left(\sum_{j=1}^{n} \gamma_t^j \right)^{1-\alpha} \right] \right\}$$

$$s.t. \quad \frac{a_{t+1}}{\rho_t} + C_t + \mathcal{S}_t = (1-\tau_t) z_t \xi_t l_t + a_t + \mathcal{T}_t,$$

$$\xi_{t+1} = \Delta\left(\frac{\mathcal{S}_t}{z_t\xi_t}\right)\xi_t \text{ and } z_{t+1} = \eta(\bar{l}_t)\phi\left(\frac{\mathcal{I}_t}{z_t\xi_t}\right) z_t.$$

Each citizen takes government policies, the interest rates, average labor, and hence z_t's, as given. Denote

$$\hat{c}_t = \frac{C_t}{z_t\xi_t}, \hat{s}_t = \frac{\mathcal{S}_t}{z_t\xi_t}, \hat{a}_t = \frac{a_t^i}{z_t\xi_t}, \hat{\mathcal{T}}_t = \frac{\mathcal{T}_t}{z_t\xi_t}.$$

From the citizens' point of view, the problem can be written as:

$$\max_{(\hat{s}_t,\hat{c}_t,l_t)_{t=0}^{\infty}} \sum_{t=0}^{\infty} \delta^t \left\{ \log \hat{c}_t + \mu \log(1-l_t) + \frac{\delta}{1-\delta}\Delta_1 \log \hat{s}_t \right\},$$

$$\Delta\left(\hat{s}_t\right) \frac{z_{t+1}}{z_t} \frac{\hat{a}_{t+1}}{\rho_t} + \hat{c}_t + \hat{s}_t = (1-\tau_t) l_t + \hat{a}_t + \hat{\mathcal{T}}_t.$$

Using the citizens' FOC with respect to savings, consumption, investment and labor we have that:

$$\begin{aligned} \Delta\left(\hat{s}_t\right) \frac{z_{t+1}}{z_t} \frac{\hat{c}_{t+1}}{\hat{c}_t} &= \delta\rho_t, \\ \mu \frac{\hat{c}_t}{1-l_t} &= 1-\tau_t, \\ \hat{s}_t + \Delta_1 \Delta\left(\hat{s}_t\right) \frac{z_{t+1}}{z_t} \frac{\hat{a}_{t+1}}{\rho_t} &= \frac{\delta}{1-\delta}\Delta_1 \hat{c}_t. \end{aligned}$$

In equilibrium, by the market clearing condition we have that $a_{t+1} = \frac{\beta_{t+1}}{n}$, where β_{t+1} is the amount of government bonds issued in period t. Hence, the last equation defines an equilibrium relationship between $\hat{s}_t$ and $\hat{c}_t$ which can be written as

$$\hat{s}_t = s^c(p_t)\hat{c}_t,$$

where $s^c(p_t) \equiv \Delta_1 \delta \left(\frac{1}{1-\delta} - \frac{\beta_{t+1}}{nC_{t+1}} \right)$.

Next, we use the resource constraint and the definitions of I_t and g_t^i to get that

$$(1+s^c(p_t))\hat{c}_t = l_t \left[1 - \frac{1}{n} \left(I_t + \sum_{i=1}^{n} g_t^i \right) \right].$$

Going back to the citizens' intra-temporal Euler equation and substituting consumption out, we have that

$$\frac{\mu}{1+s^c(p_t)} \frac{l_t \left[1 - \frac{1}{n} \left(I_t + \sum_{i=1}^{n} g_t^i \right) \right]}{1-l_t} = 1-\tau_t.$$

It follows that

$$l(p_t) = \frac{1-\tau_t}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]+1-\tau_t},$$

where $\mu_0(p_t) \equiv \frac{\mu}{1+s^c(p_t)}$. Note that,

$$1-l(p_t) = \frac{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]+1-\tau_t},$$

and, hence, consumption can be written as $C(p_t, z_t, \xi_t) = z_t \xi_t c(p_t)$, where

$$c(p_t) = \frac{(1-\tau_t)}{\mu} \frac{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]+1-\tau_t}. \tag{1}$$

Similarly, private investment can be written as $\mathcal{S}_t(p_t, z_t, \xi_t) = z_t \xi_t s(p_t)$, where

$$s(p_t) = s^c(p_t)\frac{(1-\tau_t)}{\mu} \frac{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]+1-\tau_t}. \tag{2}$$

We can also write the resource constraint of the economy as:

$$z_t \xi_t n l(p_t)\left[1-\frac{c(p_t)+s(p_t)}{l(p_t)}-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right] \geq 0.$$

Next, we define $u(c_t, l_t)$ as the citizens' utility from consumption and labor scaled by overall productivity:

$$u(\hat{c}_t, l_t) \equiv \log \hat{c}_t + \mu \log(1-l_t).$$

Substituting for $\hat{c}_t$ and l_t we get

$$u(\hat{c}_t, l_t) = \log\left(\frac{1-\tau_t}{\mu}\right) + (1+\mu)\log\left(\frac{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]+1-\tau_t}\right).$$

Finally, note that since $\hat{g}_t^i = n g_t^i l_t$, we have that

$$\omega \log\left[\left(\hat{g}_t^i\right)^{\alpha}\left(\sum_{j=1}^{n}\hat{g}_t^j\right)^{1-\alpha}\right] = \omega \log \frac{n(1-\tau_t)}{\mu_0\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^{n} g_t^i\right)\right]+1-\tau_t} + \omega \log\left[\left(g_t^i\right)^{\alpha}\left(\sum_{j=1}^{n} g_t^j\right)^{1-\alpha}\right].$$

In essence, the utility from public goods can be decomposed into two components: (i) the utility from output, and (ii) the output share of public goods.

Combining the two lines from above and collecting terms we have that

$$\begin{aligned} u^i(p_t, z_t, \xi_t) &= (1+\omega)\log z_t\xi_t + u(\hat{c}_t, l_t) + \\ &\quad +\omega \log \frac{n(1-\tau_t)}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^n g_t^i\right)\right]+1-\tau_t} + \omega \log\left[\left(g_t^i\right)^\alpha \left(\sum_{j=1}^n g_t^j\right)^{1-\alpha}\right], \end{aligned}$$

or

$$u^i(p_t, z_t, \xi_t) = (1+\omega)\log z_t\xi_t + U(p_t) + \omega \log\left[\left(g_t^i\right)^\alpha \left(\sum_{j=1}^n g_t^j\right)^{1-\alpha}\right], \tag{3}$$

where

$$U(p_t) \equiv u(\hat{c}_t, l_t) + \omega \log \frac{n(1-\tau_t)}{\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^n g_t^i\right)\right]+1-\tau_t},$$

is the citizens' indirect per-period utility function scaled by overall productivity and augmented by by the "output part" of the utility from public goods. We summarize this as

Lemma A1. *The per-period indirect utility function is given by (3) where*

$$U(p_t) = \left(\begin{array}{c} \log\frac{n^\omega}{\mu} + \log\left(1-\tau_t\right)^{1+\omega} + \\ \log\left(\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^n g_t^i\right)\right]\right)^{1+\mu} + \\ -\log\left(\mu_0(p_t)\left[1-\frac{1}{n}\left(I_t+\sum_{i=1}^n g_t^i\right)\right]+1-\tau_t\right)^{1+\mu+\omega} \end{array}\right) \tag{4}$$

We next provide the proof of Lemma 1.

2.2 Proof of Lemma 1

A citizen, whose district is in the government, has the following expected continuation value:

$$v_{\mathcal{G}}(b, z, \xi) = U(p) + \omega E \log\left[\left(g^i\right)^\alpha \left(\sum_{j=1}^n g^j\right)^{1-\alpha}\right] + \delta v(b', z', \xi).$$

Hence, the incentive compatibility constraint holds with equality iff $E\log g^i = \log g^c$. Since

$$E\log g^i|_{i\in\{1:\mathcal{G}\}} = \frac{1}{\mathcal{G}}\left(\log g + (q-1)\log g^c + (\mathcal{G}-q)\log \underline{g}\right).$$

it must be the case that $g^c = g^{\frac{1}{\mathcal{G}-q+1}}\underline{g}^{1-\frac{1}{\mathcal{G}-q+1}}$, as stated. ■

2.3 Proof of Proposition 1

Write $v_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} u(p_\tau, z_\tau, \xi_\tau)$ as:

$$v_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ (1+\omega) \log z_\tau \xi_\tau + U(p_\tau) + \omega E \log \left[\left(g_\tau^i\right)^\alpha \left(\sum_{j=1}^{n} g_\tau^j \right)^{1-\alpha} \right] \right\}.$$

Since $\log(z_{t+1}\xi_{t+1}) = \log(z_t\xi_t) + \phi_1 \log(I_t \cdot l_t) + \Delta_1 \log s_t + \eta_1 \log l_t + \log \Delta_0 \phi_0 \eta_0 n^{\phi_1}$, we have:

$$\log(z_{t+T}\xi_{t+T}) = \log(z_t\xi_t) + \sum_{\tau=t}^{t+T-1} \left[\phi_1 \log(I_\tau) + \Delta_1 \log s_\tau + (\eta_1 + \phi_1) \log l_\tau + \log \Delta_0 \phi_0 \eta_0 n^{\phi_1} \right].$$

Using this formula, we can express v_t as

$$\begin{aligned} v_t &= \frac{1+\omega}{1-\delta} \log(z_t\xi_t) + \\ &+ \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left\{ \begin{array}{c} U(p_\tau) + \frac{(1+\omega)\delta}{1-\delta} \left[\phi_1 \log(I_\tau) + \Delta_1 \log s_\tau + (\eta_1 + \phi_1) \log l_\tau + \log \Delta_0 \phi_0 \eta_0 n^{\phi_1} \right] \\ + \omega E \log \left[\left(g_\tau^i\right)^\alpha \left(\sum_{j=1}^{n} g_\tau^j \right)^{1-\alpha} \right] \end{array} \right\}, \end{aligned} \tag{5}$$

which, in turn, using the expression for $E \log g^i$ can be written as

$$v_t = \mathcal{A} \log(z_t\xi_t) + \sum_{j=t}^{\infty} \delta^{j-t} \left[\mathcal{U}(p_t) + \omega\alpha \frac{\mathcal{G}}{n} Q(\mathcal{G}, q) \log g_t + \omega(1-\alpha) \log(G_t) \right],$$

where $G_t \equiv \left(\sum_{j=1}^{n} g_t^j \right)^{1-\alpha} = g_t + (q-1) g_t^{Q(\mathcal{G},q)} \underline{g}^{(1-Q(\mathcal{G},q))} + (n-q)\underline{g}$, $\mathcal{A} = \frac{1+\omega}{1-\delta}$ and

$$\mathcal{U}(p_t) = A_0 + U(p_t) + \frac{(1+\omega)\delta}{1-\delta} \left[\phi_1 \log(I_t) + \Delta_1 \log s_t + (\eta_1 + \phi_1) \log l_t \right], \tag{6}$$

with $A_0 = \frac{(1+\omega)\delta}{1-\delta} \log\left(\Delta_0 \phi_0 \eta_0 n^{\phi_1}\right) + \alpha\omega \left(\frac{\mathcal{G}}{n} \frac{\mathcal{G}-q}{\mathcal{G}-q+1} + \frac{n-\mathcal{G}}{n} \right) \log \underline{g}$.

If we define $\mathcal{V}_t = \sum_{\tau=t}^{\infty} \delta^{\tau-t} \left[\mathcal{U}(p_\tau) + \alpha\omega \frac{\mathcal{G}}{n} Q(\mathcal{G}, q) \log g_\tau + \omega(1-\alpha) \log(G_t) \right]$ or, in a recursive form,

$$\mathcal{V}_t = \mathcal{U}(p_t) + \omega\alpha \frac{\mathcal{G}}{n} Q(\mathcal{G}, q) \log g_t + \omega(1-\alpha) \log(G_t) + \delta \mathcal{V}_{t+1}, \tag{7}$$

expression (5) can be written as $v_t = \mathcal{A} \log(z_t\xi_t) + \mathcal{V}_t$. The expected value of a citizen who is the government formateur, v_t^p, can be represented in the same. The only difference is that the proposer receives $\log g_t$ for sure instead of $E \log \left[\left(g_t^i\right)^\alpha \left(\sum_j g_t^j \right)^{1-\alpha} \right]$. We have:

$$v_t^p = \mathcal{A} \log(z_t\xi_t) + \mathcal{U}(p_t) + \alpha\omega \log g_t + \omega(1-\alpha) \log(G_t) + \delta \mathcal{V}_{t+1}. \tag{8}$$

where $\mathcal{V}_{t+1}$ is defined by (7).

The proof of the proposition now follows immediately. Since $\mathcal{A} \log(z\xi)$ is a constant, if $p(b)$ solves the problem in (7) of paper given $\mathcal{V}(b)$, then it must be an optimal reaction function given the true value function $\mathcal{A} \log(z\xi) + \mathcal{U}(p) + \alpha\omega \log g + \omega(1-\alpha) \log(G(g)) + \delta \mathcal{V}(b')$; moreover if $\mathcal{V}(b)$ satisfies (6) of the paper given $p(b)$, then $v(b, z, \xi) = \mathcal{A} \log z\xi + \mathcal{V}(b)$ is the expected value in the bargaining game. On the contrary, if $p(b)$ is an equilibrium, then we must have $v(b) = \mathcal{A} \log z\xi + \mathcal{V}(b)$ and the proposer must maximize the problem (7) of the paper. ■

2.4 Proof of Proposition 2

It is first convenient to formally define the marginal cost of public funds. Let

$$B(b,p) = \rho\left(b,p\right)\left[b + G(g) + I + T - \tau n l(p)\right].$$

From the proposer's budget constraint we have $\beta' = z\xi B(b,p)$. In order to reduce nominal debt β' by one marginal unit the required increase in taxes must be $d\tau = -\left[z\xi B_\tau(b,p)\right]^{-1}$. The net marginal reduction in utility from an increase in taxes is given by $\mathcal{V}_\tau(b,p)$, where $\mathcal{V}$ is the indirect utility function defined in the equation (6) of the paper.[6] The reduction in utility in absolute value is therefore $\frac{\mathcal{V}_\tau(b,p)}{z\xi B_\tau(b,p)}$. Moreover the marginal utility of consumption is $\frac{1}{z\xi c(p(b))}$. The marginal cost of public funds, then is: $\frac{c(p)\mathcal{V}_\tau(b,p)}{B_\tau(b,p)}$. Note that the Lagrangian multiplier λ from the problem (7) of the paper can be written as: $\lambda(b) = \frac{\mathcal{V}_\tau(b,p)}{B_\tau(b,p)}$. We conclude that the marginal cost of public funds when policies are p and the Lagrangian multiplier is λ is $MCPF(b) = \lambda(b)c(p)$.

We now prove derive the expression (8) of the paper. Consider the first order condition with respect to b' of the problem in (7) of the paper:

$$\frac{\lambda(b)Z(p)}{\rho(b,p)}\left[1 - \frac{b'}{\rho(b,p)}\frac{\partial\rho(b,p)}{\partial b'}\right] = -\delta\cdot\mathcal{V}'(b').$$

We have

$$\frac{\lambda(b)Z(p)}{\rho(b,p)}\left(1 - \varepsilon_\rho\left(b\right)\right) = -\delta\cdot\mathcal{V}'(b'), \tag{9}$$

where $\varepsilon_\rho\left(b\right)$ is the elasticity of the interest rate as defined in the paper. Let $\mathcal{V}_p(b)$ be the objective function of the problem (7) of the paper. Adding and subtracting $\alpha\omega\log\left(g(b)\right)$, we can write: $\mathcal{V}(b) = \mathcal{V}_p(b) + \alpha\omega\left(\frac{\mathcal{G}}{n}Q(\mathcal{G},q) - 1)\right)\log g(b)$. From the Envelope Theorem applied to (7), at any point of differentiability we have $\mathcal{V}_p'(b) = -\lambda\left(b\right)$, so we can write:

$$\mathcal{V}'(b) = -\lambda\left(b\right) + \alpha\omega\left(\frac{\mathcal{G}}{n}Q(\mathcal{G},q) - 1)\right)\frac{\partial g(b)/\partial b}{g(b)}. \tag{10}$$

Using (9) we have:

$$\left(1 - \varepsilon_\rho\left(b\right)\right)\frac{\lambda(b)Z(p)}{\delta\rho(p)} = \lambda\left(b'\right) - \alpha\omega\left(\frac{\mathcal{G}}{n}Q(\mathcal{G},q) - 1)\right)\frac{\partial g(b')/\partial b'}{g(b')}. \tag{11}$$

Observe now that the inter-temporal Euler equation can be written as: $\delta\rho(b,p) = c(p(b'))Z(p)/c(p)$. We can therefore write:

$$\left(1 - \varepsilon_\rho\left(b\right)\right)c(p)\cdot\lambda\left(b\right) = c(p(b'))\cdot\lambda\left(b'\right)\left[1 - \frac{1}{\lambda\left(b'\right)}\alpha\omega\left(\frac{\mathcal{G}}{n}Q(\mathcal{G},q) - 1)\right)\frac{\partial g(b')/\partial b'}{g(b')}\right]. \tag{12}$$

Define: $\Phi\left(b'\right) = 1/\left(b'\cdot\lambda\left(b'\right)\right)$, and let $\varepsilon_g(b') = \frac{\partial g(b')}{\partial b'}\frac{b'}{g(b')}$ be the elasticity of $g(b')$ with respect to b'. Using the fact that $MCPF(b) = \lambda(b)c(p)$, we obtain the result. ■

2.5 Proof of Proposition 3

If $\mathcal{G} = q = n$ and/or $\alpha = 0$, we have $\alpha\left(1 - \frac{\mathcal{G}}{n}Q(\mathcal{G},q))\right) = 0$. In this case, at the balanced growth b^* we must have: $\frac{\partial\rho(b',b^*)}{\partial b'}\frac{b^*}{\rho(b^*,b^*)} = 0$, where the derivative is evaluated at b^*. Since $MCPF(b^*) > 0$ and $\frac{\partial\rho(b',b)}{\partial b'} > 0$ at $b = b^*$, a balanced growth path is possible only if $b^* = 0$. ■

[6] The indirect utility is evaluated at the equilibrium policy $p(b)$, where b is the state. For simplicity we omit the state form the expression of the policy when it does not create confusion.

2.6 Proof of Proposition 4

In a regular balanced growth b^* we have:

$$\varepsilon_\rho(b^*) = -\alpha\omega\left(1 - \frac{\mathcal{G}}{n}Q(\mathcal{G}, q))\right)\Phi\left(b^*\right)\varepsilon_g(b^*). \tag{13}$$

where $\varepsilon_\rho(b^*) > 0$ and $\Phi\left(b^*\right) > 0$. We conclude that $\varepsilon_g(b^*) < 0$ and so $g(b)$ is decreasing in b in a neighborhood of the balanced growth level in any smooth equilibrium. Since the balanced growth path is stable, starting from b_0 in a left neighborhood, $b_{t+1} > b_t$, and so $\gamma_{t+1}/\gamma_t < y_{t+1}/y_t$ for any $t > 0$. By Lemma 1, we must have $\gamma^c_{t+1}/\gamma^c_t < y_{t+1}/y_t$, where $\gamma^c_t = g^c_t y_t$ is the dollar value of public goods allocated to the districts in the minimal winning coalition. From the first order conditions of the problem in (7) of the paper it is also easy to show that I_t is monotone in g_t, so we have $\mathcal{I}_{t+1}/\mathcal{I}_t < y_{t+1}/y_t$ as well. ■

3 Data

Data on output is obtained from BEA's National Income and Product Accounts (NIPA, Tables 7.1).[7] Data on debt is taken from the U.S. Office of Management and Budget (Table 7.1).[8] For public goods we use the U.S. Office of Management and Budget (Table 3.1).[9] Our definition of public investment comes from Federal Outlays, NIPA, Table 9.1. We have looked at two measures: "Research and Development," and "Total Investment Outlays for Major Public Physical Capital, Research and Development, and Education and Training." We conservatively use the former in our benchmark calibration. Using the latter leaves the results qualitatively unchanged. For private investment in productivity we use private sector's "Research and Development" measures from NIPA Table 5.3.

4 A model without growth

The model economy here is a close relative to the one in Section 2 of the paper. The structure of the economy is the same as in the paper, except citizens' utility is linear in consumption and there are no growth mechanisms: no private or public investment and no learning-by-doing.[10] In addition, to ease exposition, we assume that citizens benefit only from local public goods provided to their own districts.

4.1 Citizens

A continuum of infinitely-lived citizens live in n identical districts indexed by $i = 1, ..., n$. The size of the population in each district is normalized to be one. There is a single non-storable consumption good, denoted by c, which is produced using a single factor, labor, denoted by l. There is also a set of n local public goods, denoted by $\{g^i_t\}^n_{i=1}$, which can be produced from the consumption good. The variables at time t are denoted with a subscript t.

[7] Available at *http://www.bea.gov/iTable/index_ nipa.cfm*

[8] Available at *http://www.whitehouse.gov/omb/budget/Historicals)*.

[9] We use the following classification of Federal Outlays, as reported by the Office of Management and Budget: Public Goods: (i) National Deference; (ii) Education, Training, Employment and Social Services; (iii) Health, (iv) Energy; (v) National Resources and Environment; (vi) Transportation; (vii) Community and Regional Development; (viii) International Affairs; (ix) General Science, Space and Technology, (x) Agriculture; (xi) Administration of Justice; (xii) General Government; (xiii) Veterans Benefits and Services. Transfers: (i) Medicare; (ii) Income Security; (iii) Social Security; (iv) Commerse and Housing Credit. Other: Net Interest.

[10] As we have discussed in the main text, utility linear in consumption is incompatible with growth.

Citizens enjoy consumption, benefit from the local public good and supply labor. Each citizen's preferences in district i are represented by the following per period utility function:

$$u(c_t, l_t, g_t^i) = c_t - \varphi(l_t) + \omega\left(g_t^i\right), \tag{14}$$

where $\varphi(\cdot)$ captures dis-utility from working and $\omega(\cdot)$ is the public good benefit function. We assume that $\varphi(\cdot)$ is convex and increasing, while $\omega(\cdot)$ is concave and increasing. Citizens discount future per period utilities at rate δ.

There is a competitive labor market: thus, the wage rate is equal to 1. There is also a market in risk-free, one period bonds. Both citizens and the government have access to this market. The assets held by an agent in district i in period t are denoted a_t^i. The gross interest rate is denoted ρ_t: a dollar worth of bonds at time t yields ρ_t at time $t+1$.

Since citizens take fiscal policy variables as given, their maximization problem can be written as follows:

$$\max_{\{c_t, a_{t+1}, l_t\}} \sum_{t=0}^{\infty} \delta^t (c_t - \varphi(l_t) + \omega\left(g_t^i\right))$$
$$s.t. \;\; c_t + \frac{a_{t+1}}{\rho_t} = (1-\tau_t)l_t + a_t \text{ for all } t,$$

where τ_t is the tax rate and a_t is their bond holdings at the beginning of period t.

It follows that the equilibrium interest rate is $\rho = 1/\delta$ in all periods. With this interest rate households are indifferent between consuming today and consuming tomorrow. Finally, labor supply is only a function of the tax rate:

$$\varphi'(l_t) = 1 - \tau_t.$$

In what follows we write labor supply function as $l(\tau_t)$.

4.2 Public sector

The government provides local public goods. Revenues are raised by levying a proportional tax on labor income and they can be supplemented by borrowing and lending in the bond market. Government policy in any period t is described by $n+2$ dimensional vector $\{\tau_t, b_{t+1}, \{g_t^i\}_{i=1}^n\}$, where, as noted above, τ_t is the income tax rate; b_{t+1} is the amount of bonds sold and $\{g_t^i\}_{i=1}^n$ are local public goods. The government's initial debt level in period 0 is b_0. The market clearing condition implies that in all periods $a_t = \frac{b_t}{n}$.

Government policies must satisfy three constraints. First, tax revenues and net borrowing must be sufficient to cover public expenditures:

$$\frac{b_{t+1}}{\rho} - \left[b_t + \sum_{i=1}^{n} g_t^i - n\tau_t l_t\right] \geq 0. \tag{15}$$

Second, $g_t^j \geq \underline{g} \geq 0$ for all j and all t. That is, there is a minimum amount of public goods each district must receive. Third, debt is bounded: $b_t \in \left[\underline{b}, \overline{b}\right]$. The upper bound $\overline{b}$ is the maximum sustainable level of debt. It is achieved when the tax rate is set to maximize the tax revenue, each district gets $\underline{g}$, and the rest of the tax revenue is used to cover interest payments on debt. We set the lower bound $\underline{b}$ to zero.

Government policy decisions are made by a legislature consisting of representatives from each of the n districts. One citizen from each district is selected to be that district's representative.

The legislature meets at the beginning of each period. To describe how legislative decision-making works, suppose the legislature is meeting at the beginning of a period in which the current level of public debt is b_t. The process has two phases: government formation and bargaining in the government. In the first phase, one of the legislators is randomly selected to form a government, with each representative having an equal chance of being recognized. A government is a cabinet of $\mathcal{G}$ representatives and a policy platform $\{b_{t+1}, \tau_t, G_t\}$, where G_t is the aggregate amount of public goods, $G_t \equiv \sum_{i=1}^{n} g_t^i$. In the second phase, the cabinet members allocate the local public goods. The initial government formateur proposes a provisional distribution of the local public goods $\{g_t^i\}_{i=1}^n$. If the first proposal is accepted by $q \leq \mathcal{G}$ cabinet members, then it is implemented and the legislature adjourns until the beginning of the next period. At that time, the legislature meets again with the difference being that the initial level of public debt is b_{t+1}. If, on the other hand, the first proposal is not accepted, another member of the government is chosen to propose an alternative redistribution of $\{g_t^i\}_{i=1}^n$. The process continues until a proposal is approved by the cabinet. We assume that each proposal round takes a negligible amount of time. As in the paper, we define g_t and g_t^c as the amount of local public goods for, the proposer's district and the other districts in the minimal winning coalition (i.e., the subset q of the government), respectively.

We now establish a result which corresponds to Lemma 1 of the paper. Incentive compatibility for the members of the minimum winning coalition implies that the proposal must give the citizens in their districts a utility level from public goods no less than the expected value of this utility if the legislative bargaining goes to the next round:

$$\omega\left(g_t^c\right) = E\omega\left(g_t^i\right)|_{i\in\{1:\mathcal{G}\}} = \frac{1}{\mathcal{G}}\left(\omega\left(g_t\right) + (q-1)\omega\left(g_t^c\right) + (\mathcal{G}-q)\,\omega\left(\underline{g}\right)\right), \tag{16}$$

or

$$\omega\left(g_t^c\right) = \frac{1}{\mathcal{G}-q+1}\left(\omega\left(g_t\right) + (\mathcal{G}-q)\,\omega\left(\underline{g}\right)\right).$$

Naturally, when $q = \mathcal{G}$, there is a unanimity *within* the government. The other extreme is $q = 1$. In this case, the proposing legislator is in position to take the largest share of the pie, G_t, for his district.[11] We note, from the expression above, that if $\omega(0) = -\infty$, to have a well-defined problem we must set $\underline{g}$ to be strictly positive, as it is done in the paper. In what follows, to ease exposition, we follow Battaglini and Coate [2008], and assume that $\omega(\cdot)$ is a power function: $\omega(g) = g^\alpha$, $1 > \alpha > 0$, and set $\underline{g} = 0$. Hence, we have that

$$\omega\left(g_t^c\right) = \frac{1}{\mathcal{G}-q+1}\omega\left(g_t\right),$$

which, in turn, implies that the g_t^c is proportional to g_t, and, hence both g_t and g_t^c are proportional to the sum of all public goods, $G_t = g_t + (q-1)g_t^c = \left(1 + \frac{q-1}{(\mathcal{G}-q+1)^{1/\alpha}}\right) g_t$.[12]

We now establish a result which corresponds to Proposition 2 of the paper. We define $v(b_t)$ as the beginning of the period citizen's value function:

$$v(b_t) = l(\tau_t) - \frac{1}{n}G_t - \varphi(l(\tau_t)) + E\omega\left(g_t^i\right)|_{i\in\{1:n\}} + \delta v(b_t),$$

where $E\omega\left(g_t\right)|_{i\in\{1:n\}}$ is the expected public good benefit before citizens learn whether their district's legislator is chosen to the government or not:

[11] Ideally one could calibrate the values of $q, \mathcal{G}$, and $\underline{g}$ by matching the distribution of public good provision across districts.

[12] In the paper $\omega(\cdot)$ is the logarithmic function. Hence equation 16 gives rise to g_t^c which is a geometric average of g_t and $\underline{g}$. Here, it is a weighted arithmetic average.

$$
\begin{aligned}
E\omega\left(g_t\right)|_{i\in\{1:n\}} &= \frac{n-\mathcal{G}}{n}\omega\left(\underline{g}\right)+\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}}\left(\omega\left(g_t\right)+(q-1)\omega\left(g_t^c\right)+\left(\mathcal{G}-q\right)\omega\left(\underline{g}\right)\right)= \\
&= \frac{1}{n}\left(\omega\left(g_t\right)+(q-1)\omega\left(g_t^c\right)\right)=\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}\omega\left(g_t\right).
\end{aligned}
$$

It follows that the maximization problem of the proposing legislator is

$$
v^p(b_t)\equiv\max_{\tau_t,g_t,b_{t+1}}\left\{l(\tau_t)-\frac{1}{n}G_t-\varphi(l(\tau_t))+\omega_G\left(G_t\right)+\delta v(b_{t+1})\right\}
$$

$$
s.t.\quad \frac{b_{t+1}}{\rho}-\left[b_t+G_t-n\tau_t l_t\right]=0,
$$

where

$$
v(b_{t+1})=\left\{l(\tau_{t+1})-\frac{1}{n}G_{t+1}-\varphi(l(\tau_{t+1}))+\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}\omega_G\left(G_{t+1}\right)+\delta v(b_{t+2})\right\},
$$

and $\omega_G\left(G_t\right)\equiv\omega\left(g_t\right)=\left(\frac{(\mathcal{G}-q+1)^{1/\alpha}}{(\mathcal{G}-q+1)^{1/\alpha}+1}G_t\right)^{\alpha}=constant\cdot G_t^{\alpha}$.

Assuming differentiability and taking first order conditions with respect to τ_t, G_t and b_{t+1}, we have that

$$
\left[1-\varphi'(l(\tau_t))\right]l'(\tau_t)=\Lambda_t(l(\tau_t)+\tau_t l'(\tau_t)), \tag{17}
$$

$$
\left[-\frac{1}{n}+\omega_n'\left(G_{t+1}\right)\right]=\Lambda_t, \tag{18}
$$

$$
v'(b_{t+1})=-\Lambda_t, \tag{19}
$$

where Λ_t, the Lagrange multiplier on the government's budget constraint, is the marginal cost of public funds.

Next, using Envelope Theorem, we have that

$$
(v^p(b_t))'=-\Lambda_t.
$$

Finally, totally differentiating $v(b_t)$ we have that

$$
\begin{aligned}
v'(b_t) &= \left\{\left[l'(\tau_t)-\varphi'(l(\tau_t))l'(\tau_t)\frac{d\tau_t}{db_t}\right]+\left[-\frac{1}{n}+\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}\omega_n'\left(G_t\right)\right]\frac{dG_t}{db_t}+\delta v'(b_{t+1})\frac{db_{t+1}}{db_t}\right\}= \\
&= (v^p(b_t))'+\left[\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}-1\right]\omega_n'\left(G_t\right)\frac{dG_t}{db_t}=-\Lambda_t+\left[\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}-1\right]\omega_n'\left(G_t\right)\frac{dG_t}{db_t},
\end{aligned}
$$

which is the analog of equation (10) in the proof of Proposition 2. It follows,

$$
\Lambda_t=\Lambda_{t+1}-\left[\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}-1\right]\omega_n'\left(G_{t+1}\right)\frac{dG_{t+1}}{db_{t+1}},
$$

which is the analog of the main result of Proposition 2.

When there are political economy distortions, i.e., when $\frac{\mathcal{G}}{n}\frac{1}{\mathcal{G}-q+1}\neq 1$, for this economy to reach an interior steady state it must be the case that $\frac{dG_t}{db_t}=0$. This, cannot happen.[13] We conclude that in this economy debt is ever-growing and converges to $\bar{b}$. Provision of public goods collapses and taxes rise to the revenue maximizing level.

[13] Indeed, since the Lagrange multiplier is always positive, FOC (18) and (19) imply that if $\frac{dG_t}{db_t}=0$ then also $\frac{db_{t+1}}{db_t}=0$. Similarly, FOC (18) and (17) imply that if $\frac{dG_t}{db_t}=0$ then also $\frac{d\tau_t}{db_t}=0$. But totally differentiating the budget constraint makes it clear that the policy functions cannot all have zero derivative.

4.3 Small open economy considerations

The same dynamics of fiscal policy variables can be obtained for a small open economy. Below we introduce our political economy set up of fiscal policy determination into a standard small economy model. Suppose both citizens and the government have unrestricted access to world capital markets that supplies capital at the interest rate of $\rho^* = 1/\delta$. There is no default.[14] The rest of the model is identical to the one above. Households start with assets a_0 and maximize

$$\max_{\{c_t, a_{t+1}, l_t\}} \sum_{t=0}^{\infty} \delta^t (c_t - \varphi(l_t) + \omega\left(g_t^i\right))$$
$$s.t. \;\; c_t + \frac{a_{t+1}}{\rho^*} = (1-\tau_t) l_t + a_t \text{ for all } t.$$

Recall that with the linear utility in consumption citizens do not care about the timing of their consumption. Hence, without loss of generality we can set private lending and borrowing to zero,[15] and consumption function is simply $c(\tau_t) = (1-\tau_t) l(\tau_t) = l(\tau_t) - \tau_t l(\tau_t)$. [16]

It is immediate to show that the bargaining procedure from the previous section implies that the government's maximization problem is

$$v^p(b_t) = \max_{\tau_t, g_t, b_{t+1}} \{l(\tau_t) - \tau_t l(\tau_t) - \varphi(l(\tau_t)) + \omega_G\left(G_t\right) + \delta v(b_{t+1})\}$$
$$s.t. \;\; \frac{b_{t+1}}{\rho^*} - [b_t + G_t - n\tau_t l_t] = 0,$$

where

$$v(b_{t+1}) = \left\{ l(\tau_t) - \tau_t l(\tau_t) - \varphi(l(\tau_t)) + \frac{\mathcal{G}}{n} \frac{1}{\mathcal{G} - q + 1} \omega_G\left(G_{t+1}\right) + \delta v(b_{t+2}) \right\},$$

which yields, just as above:

$$\Lambda_t = \Lambda_{t+1} - \left[\frac{\mathcal{G}}{n} \frac{1}{\mathcal{G} - q + 1} - 1 \right] \omega_n'\left(G_{t+1}\right) \frac{dG_{t+1}}{db_{t+1}}.$$

Hence, in a small open economy with a government that faces inelastic interest rates, fiscal policy variables exhibit the same qualitative patterns as in a closed one: debt keeps rising, public good provision collapses and taxes rise to the revenue maximizing levels.

In the small open economy version of the model presented above the interest rate is assumed constant over time and inelastic with respect to debt. Of course, there are mechanical ways of forcing public debt level to converge to an interior steady state even for a small open economy. E.g., we could assume exogenously that the interest rate rises with the debt level (capturing unmodeled default fears etc...) or that they are real costs of adjusting debt.

We conjecture that in large open economies (those that do affect the world's interest rate) the dynamics of fiscal policy will be qualitatively similar to the one in our benchmark model. In particular, our analysis in the paper and in this appendix suggests that the smaller is the elasticity

[14] Of course, without the risk of default, $1/\delta$ is the only interest rate at which citizens' problem does not imply either continuos accumulation or de-cumulation of assets.

[15] Or, we can assume, that households keep their asset holding constant and, depending on the sign of a_0, either pay or collect interest on it.

[16] Note that in this economy market clearing does not imply that $a_t = \frac{b_t}{n}$. Instead, we have that net exports are defined as $nx_t = \frac{a_{t+1} - b_{t+1}}{\rho} - (a_t - b_t)$ and $c_t + G_t + nx_t = l_t$.

of the interest rate with respect to a given country's fiscal policy, the larger that country's public debt will be. Vice-versa, the larger is this elasticity, the smaller the debt level will be. It is tempting to conclude that (1) the decline in the elasticity of interest rate associated with the liberalization of international financial markets in the past decades was a driver in the rise of public debt; and (2) within same monetary union, it is more likely that a small country like Greece (that does not control market interest rates) rather than a large country like Germany (whose policies affect the interest rates) will find itself on an unsustainable path of debt growth. We leave these questions for future research.

5 Robustness: Long run behavior of the economy

Below we present figures to illustrate the dynamics of the economy for each case presented in Table 2 of the paper. For each parameterization, the corresponding figure presents the evolution of the economy starting from no debt. To make figures readable, we present the first eighty periods of the transition path. By then, in all cases the economy either has converged or very close the balanced growth path values reported in Table 2.

As the figures show, in all cases there is the shrinking government effect, accompanied by the decline in labor supply and productivity growth.

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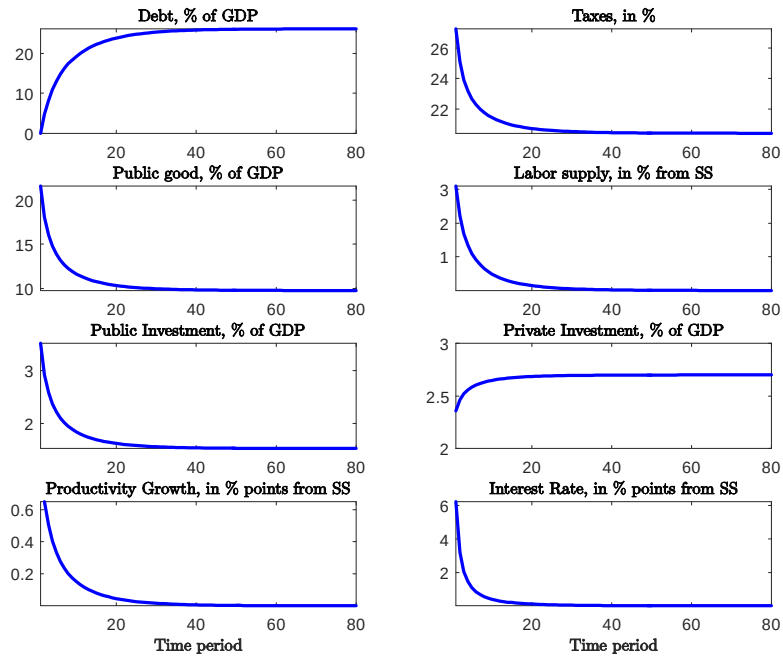

Figure 1: High δ

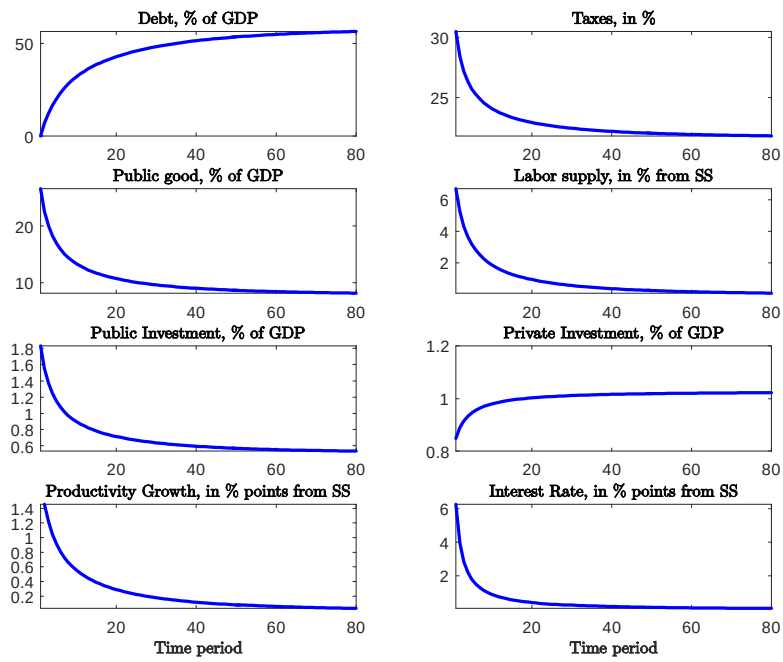

Figure 2: Low δ

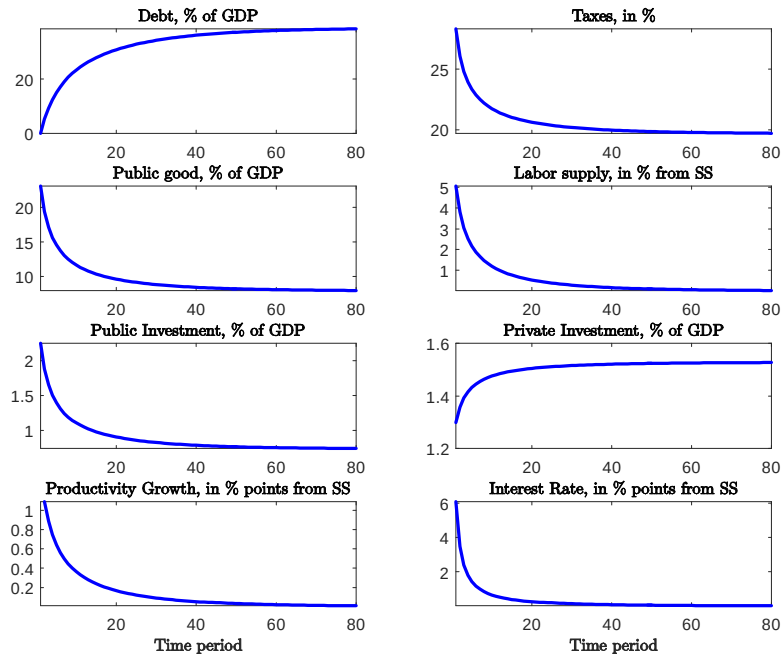

Figure 3: High μ

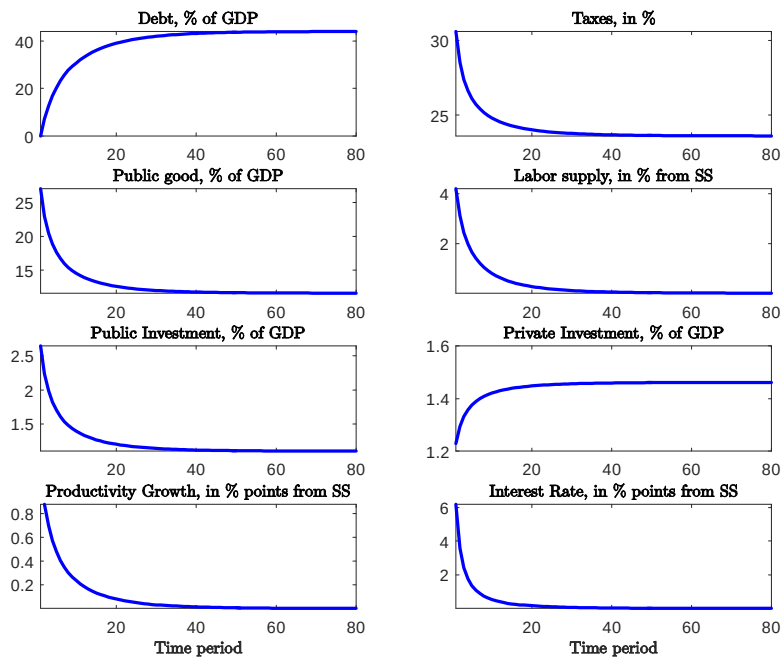

Figure 4: Low μ

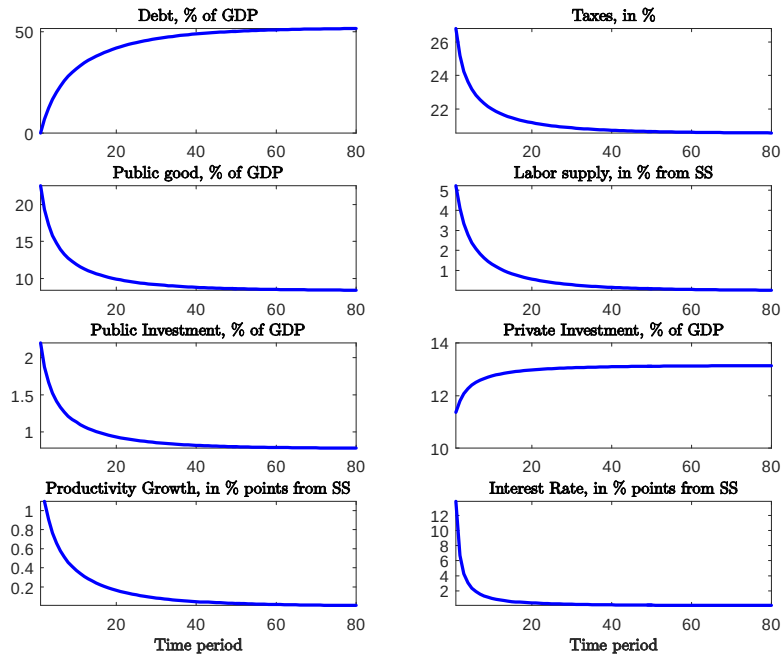

Figure 5: High Δ_1

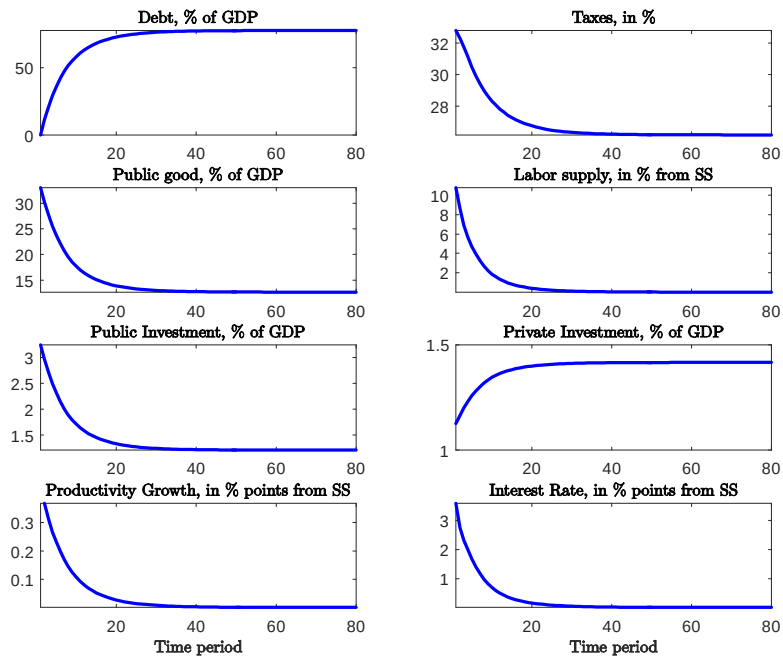

Figure 6: Low ϕ_1

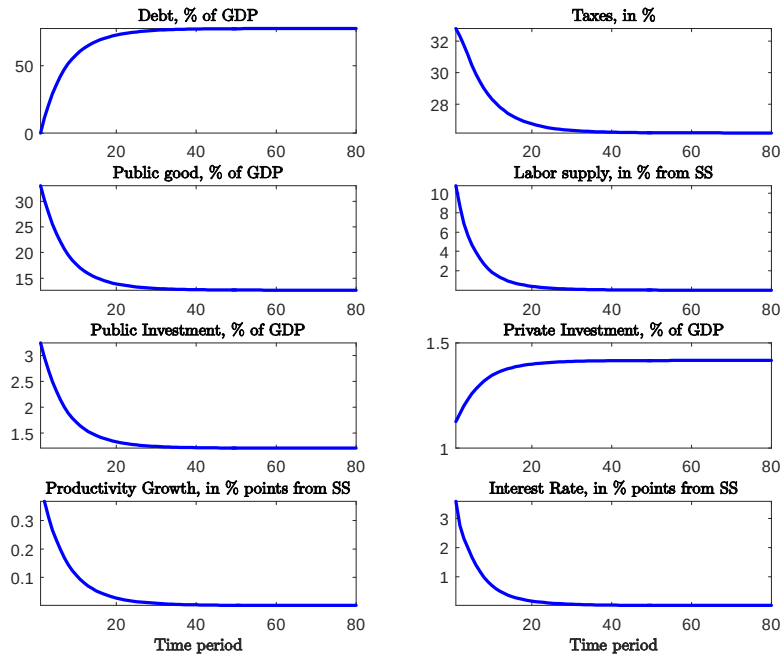

Figure 7: Low η_1

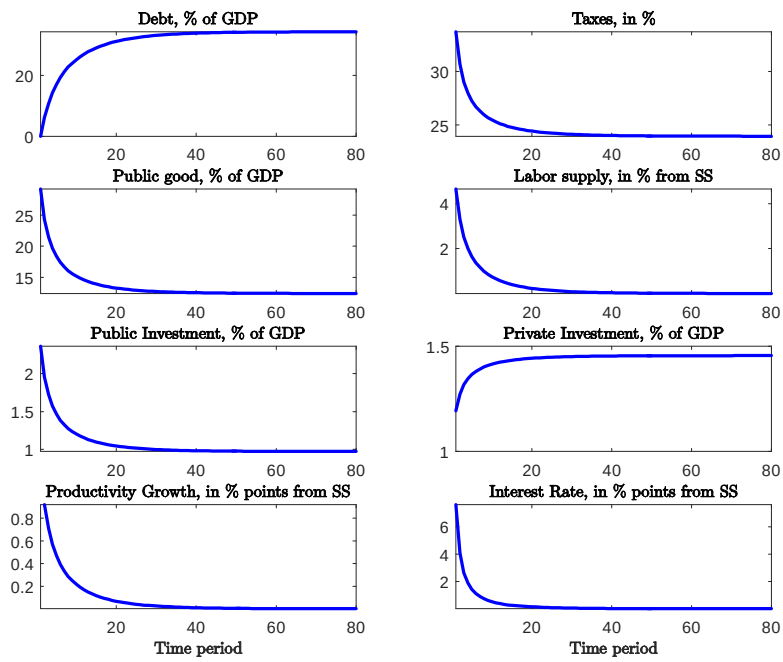

Figure 8: High ω

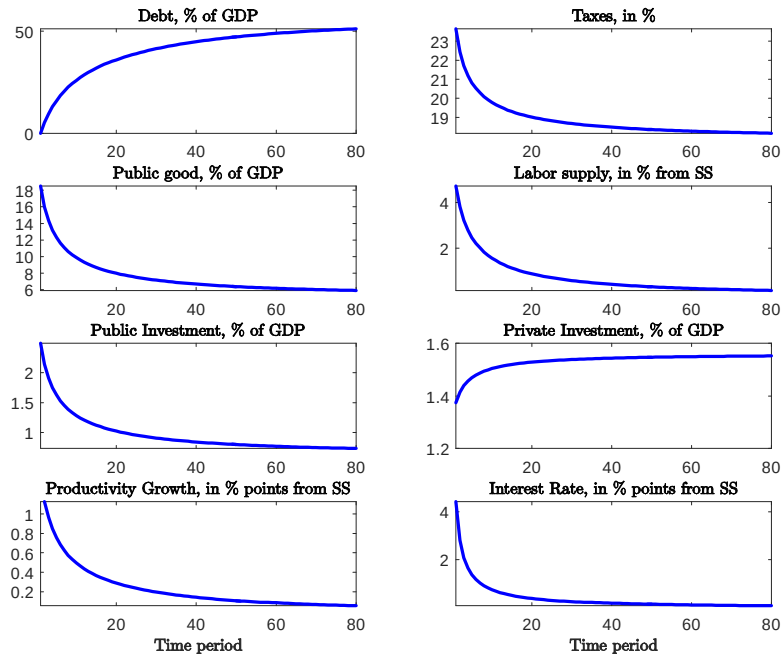

Figure 9: Low ω

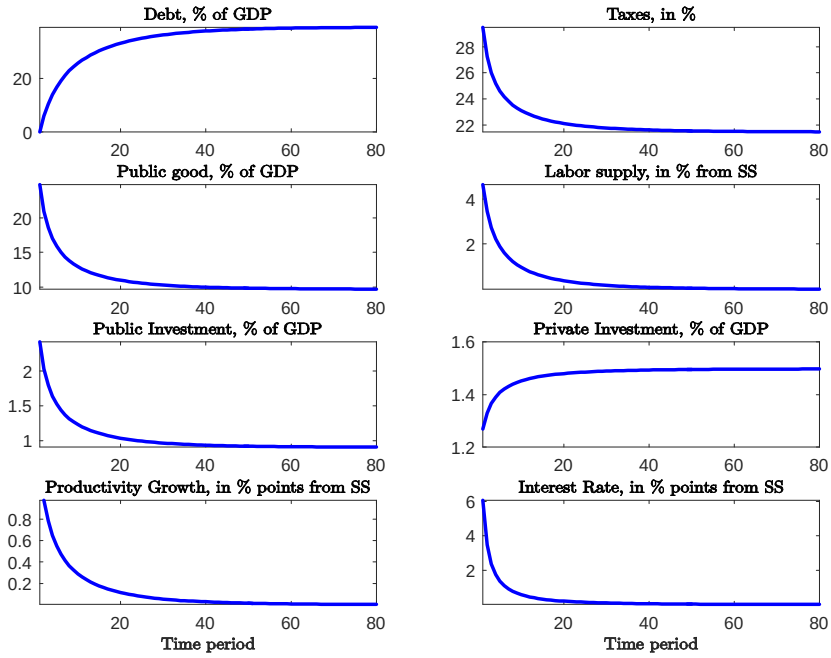

Figure 10: High q

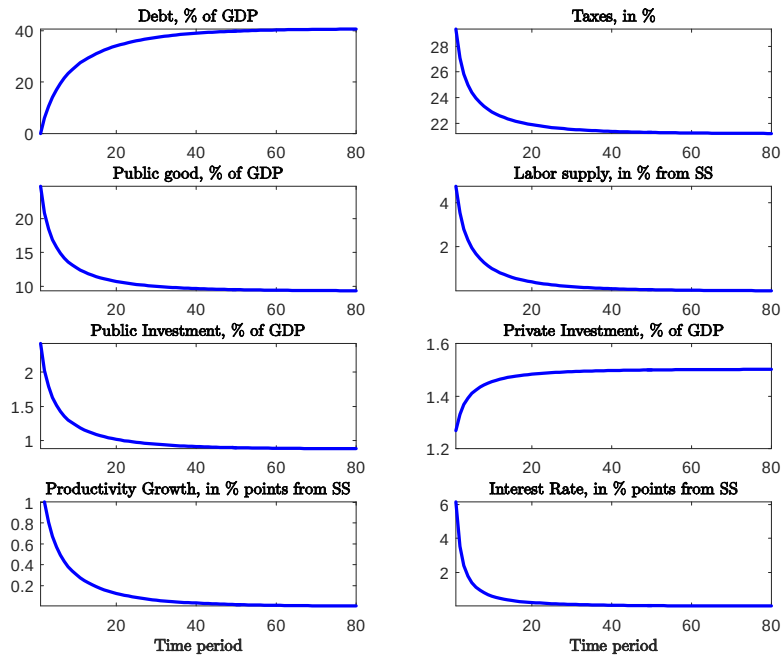

Figure 11: Low q

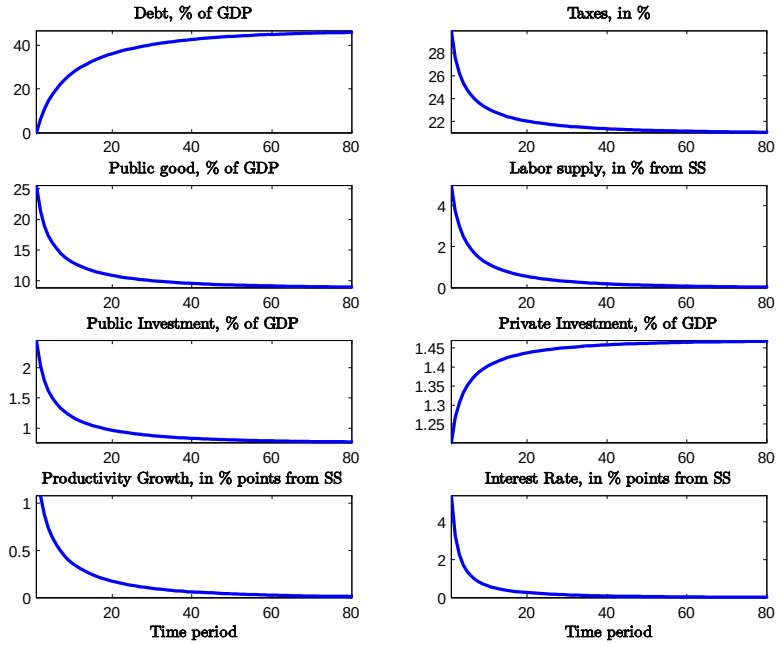

Figure 12: High $\underline{g}$

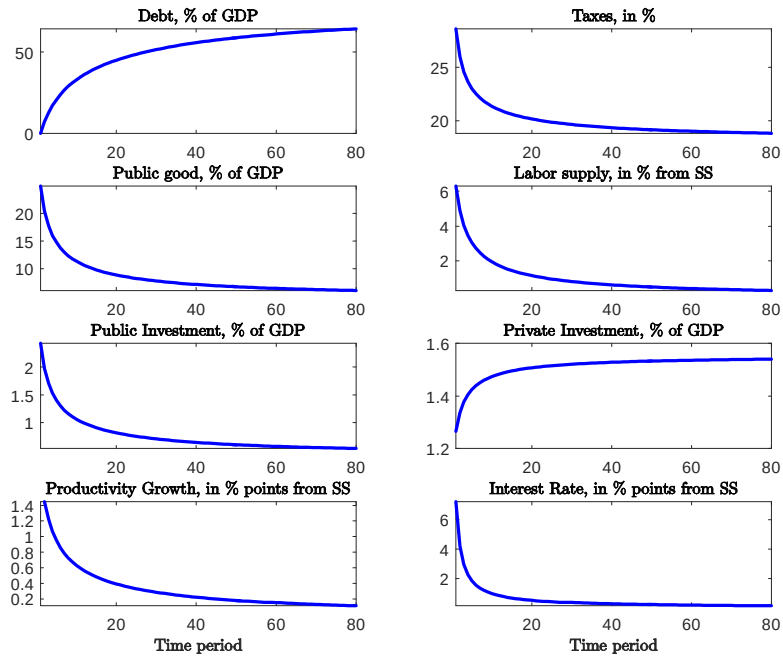

Figure 13: High α

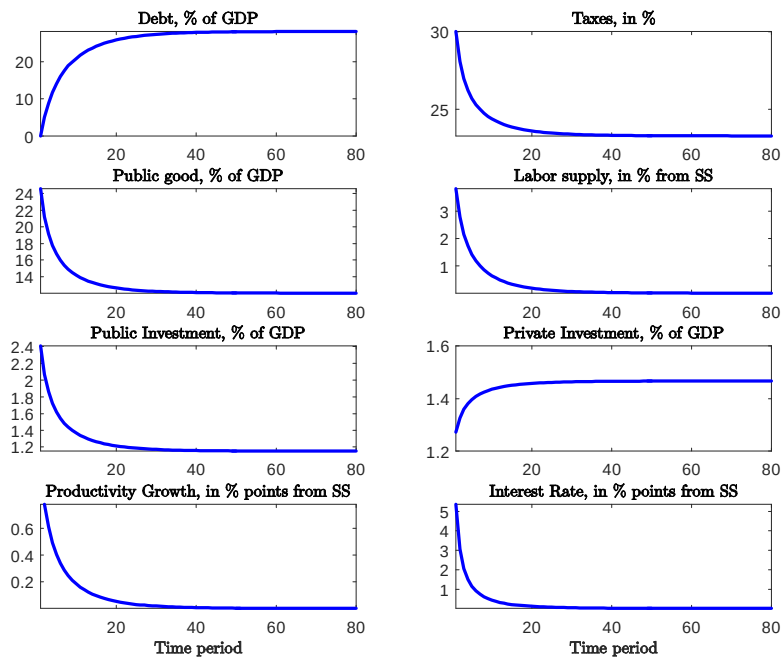

Figure 14: Low α

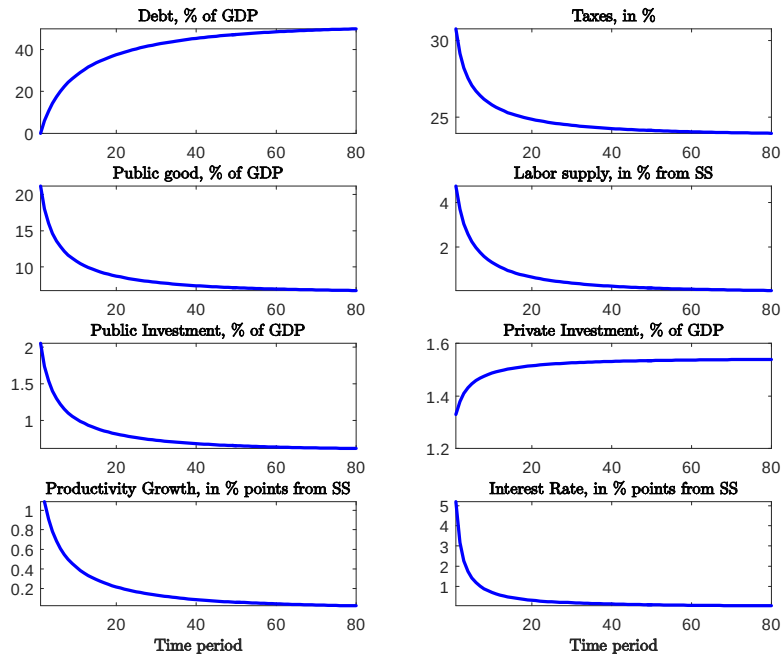

Figure 15: High transfers

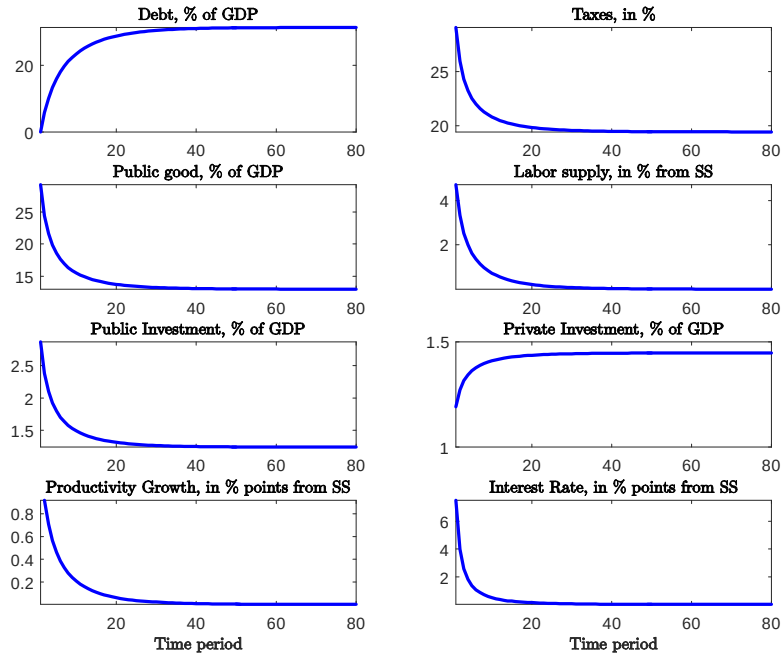

Figure 16: Low transfers