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Coalition Formation in Legislative Bargaining*

Abstract

We propose a new model of legislative bargaining in which coalitions may have different values, reflecting the fact that the policies they can pursue are constrained by the identity of the coalition members. In the model, a formateur picks a coalition and negotiates for the allocation of the surplus it is expected to generate. The formateur is free to change coalitions to seek better deals with other coalitions, but she may lose her status if bargaining breaks down, in which case a new formateur is chosen. We show that as the delay between offers goes to zero, the equilibrium allocation converges to a generalized version of a Nash Bargaining Solution in which –in contrast to the standard solution– the coalition is endogenous and determined by the relative coalitional values. A form of the hold-up problem specific to these bargaining games contributes to generate significant inefficiencies in the selection of the equilibrium coalition. We show that the model helps rationalize well known empirical facts that are in conflict with the predictions of standard non-cooperative models of bargaining: the absence of significant (or even positive) premia in ministerial allocations for formateurs and their parties; the occurrence of supermajorities; and delays in reaching agreements.

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1 Introduction

In most parliamentary democracies, public policies are not decided in elections, but are instead the outcome of elaborate bargaining processes in the Parliament. Elections often do not even determine the identity of the governing coalition, which may indeed be difficult to predict on the basis of the electoral outcome alone. After the 2017 German election, a coalition of the Christian Democrats (CDU/CSU) with the Social Democrats (SPD) on the left was formed only after a failed attempt to form a coalition between the CDU, the Free Democratic Party (FDP) and the Greens. After the 2018 Italian elections, the 5 Star Movement contemplated the formation of a coalition with the Democratic Party (DP) on the left, before converging to the Northern League on the right. Predicting the outcome of legislative bargaining is generally hard because it is not just about dividing surplus within some minimal winning coalition. For the CDU/CSU, forming a government with the SPD is more than just the number of ministers that need to be conceded to the SPD compared to the FDP or the Greens: it is also about what can be achieved in each coalition. Understanding how these two often conflicting goals interact is clearly central in understanding how parliamentary democracies work.

In this paper we present a new theory of legislative bargaining in which formateurs have to reconcile the need to form the most productive coalition with the desire to maximize the share of output that they capture. The key assumptions underlying our analysis are that coalitions are heterogeneous in terms of the surplus that they are expected to generate for the legislators; and that formateurs can search for the optimal coalition, free to change the target coalition if they can't reach an agreement. Are there general lessons to learn on which types of coalitions will form? Will the coalition generating the highest surplus emerge in equilibrium? If this is not the case, will at least the bargaining process avoid that the worst coalition emerges? Naturally, these are general questions that do not arise only in legislative bargaining problems, but in all environments in which the efficiency of the coalition formation process is an issue. While we focus on legislative bargaining, the bargaining model we develop in this paper can be applied to any such situations.

The traditional literature on legislative bargaining a' la Baron and Ferejohn [1989] has focused on purely redistributive environments in which all coalitions generate the same surplus, thus de-emphasizing the issue of efficiency. The theoretical literature on non-cooperative bargaining with coalitions of heterogeneous values, on the other hand, has followed the tradition in cooperative games, studying superadditive environments in which the grand coalition including all players is the most efficient, and focusing on when this grand coalition with all players emerges in equilibrium. The environments studied in these papers are best suited to model environmental or peace negotiations, where inclusive outcomes are desirable; they are less suitable for legislative problems, where redistributive considerations are important. In the legislative context, the grand coalition is typically not the most efficient and the more interesting questions are instead how inefficient the equilibria could be, who is in the majority, and who is out. Besides assuming superadditive environments and focusing on inclusive equilibria, moreover, these models adopt bargaining protocols that, while direct extensions of Rubinstein's iconic approach, may appear unnatural in many applications, including legislative bargaining.

The bargaining model that we propose attempts to fill the missing gap between these two literatures, extending the basic model of legislative bargaining by allowing for coalitions with heterogeneous values without superadditivity, and with a simple bargaining protocol designed to model actual legislative processes.

In our model, bargaining starts with the appointment of a formateur in charge of selecting a majority and allocating within its members the surplus it is expected to generate (for example by selecting ministerial appointments). Coalitions are heterogeneous because their feasible policy space, and thus the surplus that they generate, depends on their members. Each possible coalition C generates a value $V(C)$ to be distributed; once a coalition is selected the formateur negotiates with their members on how to allocate it with a process of alternating proposals. A distinctive feature of this process in our model is that, even after the start of internal negotiations, the formateur is not bound to a coalition, s/he can always turn to a different coalition if optimal. This captures the specific role played by the formateur: s/he is not just bargaining on an allocation within a coalition, but primarily in *search* of a coalition, thus free to halt negotiations with a stubborn coalitional partner and turn to another. We assume that protracting negotiation is costly because, in addition to intertemporal discounting, at every round there is a probability of bargaining breakdown as in Binmore, Rubinstein and Wolinsky [1986]. A bargaining breakdown leads to either a new election or to the nomination of a new formateur (perhaps after a new election). If a new formateur is selected, the process restart with the new formateur. Bargaining ends when a coalition reaches an agreement or (if it is a possibility) there is a bargaining breakdown that leads to a new election with exogenous status quo.

The bargaining protocol described above includes as a special case the well known protocol in Baron and Ferejohn [1989], where it is assumed that the formateur makes a take-it-or leave-it (TIOLI) offer to the coalition partners, after which there is a bargaining breakdown and a change in formateur. If, in our model, we assume that the probability p of a bargaining breakdown is 1, then we have a version of Baron and Ferejohn [1989] with a deterministic rotating order of formateurs.[1] If instead we assume $p < 1$, then we have a model in which, except in the unlikely

[1] In Baron and Ferejohn [1989], formateurs are randomly selected with replacement. We present a version of our model with the same process of random selection in Section 6.2, both for the case with TIOLI offers as in Baron

event of an exogenous breakdown, the decision to change coalitions is left to the formateur: and so we have meaningful intracoalitional negotiations.

We first characterize the equilibrium of the bargaining game between formateur and the coalitions assuming exogenous continuation values in case of breakdown, as traditionally assumed in the literature.[2] We show that bargaining leads to a unique stationary equilibrium in which an inefficient coalition is generally selected. The equilibrium choice of coalition depends on the probability of bargaining breakdowns, the size of the coalition and its associated surplus. The inefficiency is due to a form of the *hold-up problem* that limits the formateur's bargaining power, due to the fact that s/he cannot credibly commit to switch to any other coalition. The threat of being held up does not lead to underinvestment as in the standard hold-up problem, but to an inefficient choice of coalition (in terms of net total surplus). As the delay between offers goes to zero, the equilibrium allocation converges to a generalized version of the Nash bargaining solution in which each coalition member receives its outside option plus an equal share of surplus net of reservation utilities. This is the same allocation as in the classic Nash bargaining solution: the difference is that in the n-person Nash bargaining solution the coalition is assumed to be comprised by all players (or some other exogenous coalition), while in our model it is *endogenously* (and inefficiently) determined.

To understand the types of coalitions that may emerge in equilibrium (minimal winning or super majorities, for example) and how surplus is allocated within them, we fully endogenize the reservation utilities by assuming that a bargaining breakdown by some formateur is followed by an attempt by some other formateur. We show that equilibria of this fully recursive version of the game are very different from the equilibria emerging in the existing non cooperative models a' la Baron and Ferejohn [1989]. Multiple stationary equilibria with different welfare properties and different payoff allocations typically exist, even if the core is empty. This is in conflict with the finding of Baron and Ferejohn [1989], where multiple stationary equilibria are possible but they all lead to unique values for the players. Multiplicity reflects the complexity of the strategic interaction in the model in which both the identity of the coalition and the allocation are part of the outcome. We show, however, that multiplicity does not lead to indeterminate behavior, but to a characterization that is tight enough for welfare and positive analysis. This allows us to study the condition under which an efficient equilibrium is feasible. Depending on the parameters, inefficient equilibria can coexist with efficient equilibria or be the unique outcome. Under some conditions the inefficiency can be so bad that *the least efficient coalition* is chosen in equilibrium: such inefficient equilibria always exist if the value of the feasible coalitions are sufficiently similar.

and Ferejohn [1989] and with intracoalitional bargaining as in our more general model.

[2] See Osborne and Rubinstein [1990, ch.4] for a survey of models with the possibility of bargaining breakdown.

Besides providing a new perspective on an old problem, our model provides a unified framework to explain empirical evidence that has been seen to conflict with standard models of legislative bargaining. Existing models of legislative bargaining a' la Baron and Ferejohn [1989] predict a very large formateur's premium in terms of the share of captured surplus (i.e. ministerial cabinets).[3] Our model may help to explain why we do not observe such large formateur's premia and indeed why the real benefit of being a formateur is not in the *share* s/he appropriates within a coalition, but in the *choice* of coalition. Our model also explains why we might observe delays in bargaining even in the absence of asymmetric information, or shocks during the bargaining process. Finally, our model explains why we might observe supermajority even if the supermajorities are not much more efficient than the minimal winning coalitions. The idea that legislative bargaining leads to minimal winning coalitions has been at the center of theoretical models since the classic works by Riker [1962]. It is however the case that, at least since World War II, supermajorities have been quite common in Western Europe.[4]

The organization of the remainder of the paper is as follows. We discuss related literature in the next subsection. Section 2 outlines the model. Section 3 presents the characterization of the equilibrium with exogenous outside options, highlighting the connection with the Nash bargaining solution. The characterization in Section 3 is used in Section 4 to endogenize the outside options in a fully recursive model in which a bargaining breakdown is followed by the appointment of a new formateur. Section 5 presents the positive analysis of the model: the size of the formateur's premium, the emergence of super majorities, and strategic delays. Section 6 presents extensions and variations of the basic analysis: comparing the results of the basic model with intracoalitional bargaining with a model in which the formateur makes take-it-or-leave-it offers; assuming random recognition of formateurs and proposers; and allowing for externalities. Section 7 concludes.

1.1 Related literature

The literature on legislative bargaining has traditionally focused attention on the study of divide the dollar games in which all winning coalitions divide a "pie" of fixed size. The standard reference in this body of work is Baron and Ferejohn [1989], one of the first papers to propose an elegant extension of Rubinstein's model of bilateral bargaining to the multilateral case. In this model a legislator is randomly selected to propose a division of one dollar; if the proposal is approved by a

[3] See for example, Browne and Franklin [1973], Laver and Schoffield [1990], Warwick and Druckman [2001], and Laver [1998] for surveys. There is indeed evidence of *negative* formateurs' premia relative to their size (Warwick and Druckman [2006]). These findings have been interpreted as major failures of noncooperative models (Laver [1998]).

[4] See Druckmand and Thies (2002) and Laver and Schoffield (1990), among others. Note that we are referring to supermajorities that are not unanimous (that are extremely rare).

majority of legislators, it is implemented; if it is not approved, then another legislator is randomly selected with replacement to propose another division of the dollar and the game repeats.[5]

Negotiations in which coalitions have heterogeneous values have been studied, in the larger context of non-cooperative theories of multilateral bargaining, by Chatterjee et al. [1993], Okada [1996], Seidmann and Winter [1998], Compte and Jehiel [2010] among others.[6] These papers follow the tradition in cooperative games to focus on superadditive values, i.e. environments in which the coalition of all players is always more productive than smaller coalitions. They moreover focus on the existence of equilibria in which the coalition including all players is formed. In terms of bargaining protocol, Chatterjee et al. [1993] and Seidmann and Winter [1996] assume a first rejector-proposes rule, according to which the first player to reject a proposer's offer becomes the new proposer. This procedure is a straightforward extension of Rubinstein's approach in a bilateral context, but less natural in a multilateral context and at odds with the practice to give proposal power to a formateur who is in charge of testing, potentially, more than one possible coalition. Okada [1996] and Compte and Jehiel [2010] instead consider a protocol in which, if the selected coalition does not unanimously approve the formateur's proposal, the formateur is automatically removed and a new formateur is randomly selected with replacement from the floor (even if the subset of the coalition approving the proposal is a proper majority). This again does not capture the role of the formateur after a rejection, who may decide to continue negotiations, perhaps with a subset of the initial coalition or an altogether different coalition.

To develop our theory with heterogeneous coalitions and endogenous coalition selection, we build on a model proposed by Osborne and Rubinstein [1990], who consider a simple bargaining environment with heterogeneous coalitions but without superadditivity.[7] These authors consider an environment with 3 players, one seller and 2 potential buyers with different valuations for the good sold by the seller. In this case, the only feasible coalitions consist of the seller and one of the buyers. As in our model, the seller can switch partners after an offer is rejected and before making a new offer. This game has a unique subgame perfect equilibrium in which the efficient allocation is always reached (i.e. the good is sold to the buyer with the highest valuation). We instead attempt to model a setting of multilateral bargaining with n players and more general coalition structures. While it is natural in Osborne and Rubinstein [1990] to assume that the

[5] The basic structure of the model has been extended to consider alternative bargaining protocols (Morelli [1999], Seidman et al. [2007], Ali et al. [2019]) and others; to consider richer policy spaces (Austen Smith and Banks [1988], Baron [1991]); to allow for imperfect information (Ali [2006], Baliga and Serrano [1995]); and to endogenize the proposal power (Austen-Smith and Banks [1988], Baron and Diermeier [2001], Yildirim [2007], Ali [2015]).

[6] Gul [1989] and Hart and Mas-Colell [1996] provide microfoundations of the Shapley Value. Ray and Vohra [2001] and Okada [2010] consider games in which multiple coalitions can simultaneously form and play against each other in a game.

[7] See Osborne and Rubinstein [1990, ch. 9.4]. The same bargaining procedure is adopted in an earlier model by Wilson [1984] and Binmore [1985].

player who can choose the partner is constant and exogenously given (the seller, in their model), it is more natural in our model to allow the formateur's identity to change over the course of the negotiation if an agreement is not reached. The key differences with their work, therefore, are that we allow for more general coalitional structure, and for the formateur to be replaced by another formateur if s/he fails to form a coalition, thus making reservation utilities endogenous. These differences have important implications for the strategic analysis. For example, while Osborne and Rubinstein's [1990] model has a unique equilibrium in which the efficient coalition always forms, in our model the equilibrium can be inefficient and, with endogenous reservation utilities, multiple stationary equilibria are possible. Osborne and Rubinstein [1990], moreover, does not study the relationship between the equilibrium and the Nash bargaining solution.[8]

A model of legislative bargaining with heterogeneous coalitions is also presented by Diermeier et al. [2003]. In this model, a formateur selects a coalition ex ante and then negotiates with its members in a process a' la Baron and Ferejohn with unanimity, without the possibility of shifting coalitions. Coalitions generate levels of surplus that depend on their size and on a stochastic state variable that is assumed to change during the negotiations and is unknown when the formateur selects the coalition. The equilibrium coalition depends on the legislators' patience since patient legislators are more willing to embrace larger coalitions (that are assumed to be more durable) even if they are harder to form in the bargaining stage. A feature of this model is that distribution and efficiency considerations are independent of each other. This follows from the fact that in the bargaining stage unanimity is required and the formateur cannot switch coalition once bargaining has started, even if the state variable has changed. The authors use data from nine West European countries over the period 1947-1999 to structurally estimate the legislators discount factor and the degree to which the size of a coalition increases its durability.

The way we model the bargaining protocol is an important component of our theory, intimately connected with how coalitions are selected and thus with the equilibrium impact of their heterogeneity in values. With the exception of the model in Osborne and Rubinstein [1990], the selection of the coalition and the allocation of the rents are collapsed in one step in the legislative protocols described above: a take-it-or-leave-it offer made by a proposer; if the offer is not accepted, then automatically a new proposer is selected or the first rejector becomes proposer. In our model, we explicitly model the negotiation within the coalition and we allow the formateur to switch coalitions in the midst of negotiations. As we discuss in greater detail in Sections 6.1 and 6.2, where we compare our bargaining protocol with the standard take-it-or-leave-it protocol, this has important theoretical implications. In a world in which a take-it-or-leave-it offer is followed

[8] The relationship between the equilibrium and the Nash Bargaining solution is also not studied in Wilson [1984] and Binmore [1985]. Both of these papers and Osborne and Rubinstein [1990] assume a regular discount factor δ a' la Rubinstein [1982], without bargaining breakdowns and an explicit modelling of outside options.

by the mechanical selection of another formateur, reservation utilities depend on the exogenous recognition probabilities, but they do not allow the formateur to react to a no agreement decision. When negotiations are allowed to continue, reservation utilities depends on whether a threat to switch coalitions is *credible* and whether negotiations are expected to lead to an agreement or not. Understanding the connection between these two elements, the heterogeneity of coalitions and the formateur's choice of coalitions, is key to understanding the logic behind our model and our results.[9]

As we noted, our model contributes in explaining some empirical "anomalies" from the standard model whose study has characterized the empirical literature on legislative bargaining. A number of important works have attempted to provide theories to explain them. In the context of a purely distributive model, Morelli [1999] has provided a demand theory of legislative bargaining that can explain why the proposer does not receive large premia. Baron and Diermeier [2001], Seidmann et al. [2007] have provided bargaining theories with a formateur in which supermajorities can emerge in equilibrium.[10] Acharia and Ortner [2013], Ali [2006] and Diermeier et al. [2003], among others, provide models with strategic delay in multilateral bargaining models.[11] While all these papers contribute to understanding different specific aspects of strategic bargaining, an advantage of our theory is that it provides a coherent and unified potential explanation of all these empirical "anomalies."

2 Model

We consider a model in which n parties bargain over the formation of a government. The set of parties is denoted $N = \{1, ..., n\}$. A government is formed if a qualified majority approves it. The set of qualified majorities is denoted $\mathcal{C}$. We say that a coalition C is a minimal winning coalition if $C \in \mathcal{C}$ and for any $C' \subset C$, then $C' \notin \mathcal{C}$. The set of minimal winning coalitions is denoted $\mathcal{M}$, its complement in $\mathcal{C}$, $\mathcal{S}$, is the set of supermajorities. The set of qualified majorities and minimal winning coalitions to which party i belongs are denoted, respectively, $\mathcal{C}_i$ and $\mathcal{M}_i$.

A government is defined by the supporting winning coalition and an internal allocation of the surplus generated by the government. Coalitions are not necessarily equivalent in terms of

[9] This feature of negotiations is not important only in legislative bargaining. The importance of separating between intra-coalition bargaining (i.e. how surplus in a coalition is divided) and the competitive pressure from other coalitions is also highlighted by the experimental evidence presented in Brown, Falk and Fehr [2004]. They find that markets resemble a collection of bilateral trading islands in which payoffs are split equitably between coalition partners, rather than a competitive market.

[10] Supermajorities can also be explained in the Baron and Ferejohn [1989] model under the assumption of deliberations under an "open rule." Even in this case however, the size of the coalition converges to the size of a minimal winning coalitions as the number of legislators is sufficiently large.

[11] See Section 5.3 for a more complete description of the literature.

generated surplus. We assume that the surplus generated by a coalition $C \in \mathcal{C}$ is $V(C) \geq 0$ bounded above by a finite $\overline{V} = \max_{C \in \mathcal{C}} V(C)$ and below by nonnegative $\underline{V} = \min_{C \in \mathcal{C}} V(C)$ (and it is $V(C) = 0$ for $C \notin \mathcal{C}$). We assume that the members of the coalition can share the surplus in any way they want. A feasible allocation in C is $\mathbf{x} \in X(C)$, where:

$$X(C) := \left\{ \mathbf{x} \in R^n \middle| x_i \geq 0, \ \sum\nolimits_{i \in N} x_i \leq V(C) \right\}.$$

where $n(C)$ is the number of parties in coalition C. Parties evaluate the governments according to the surplus they receive, so $\{C, \mathbf{x}\} \succeq_i \{C', \mathbf{x}'\}$ if and only if $x_i \geq x'_i$. The parties that are left outside the coalition may receive transfers from the coalition, but they otherwise receive zero.

In the baseline model, bargaining is as follows. We assume that there is a set $\mathcal{T} = \{0, ..., T\} \subseteq N \cup \{0\}$ of potential formateurs who are ordered by a priority list. We assume that the set of formateurs is regular, in the sense that for all $C \in \mathcal{C}$, the set $C \cap \mathcal{T}$ is not empty. The list of formateurs may depend on the result of the election: indeed, typically parties are recognized as formateurs in order of their electoral size. Since we do not model the electoral stage, we take this order as exogenous.

At time $t = 0$ the first formateur f^0 is recognized and makes a proposal $\mathbf{x} \in X(C)$ to a coalition $C \in \mathcal{C}$ with $f^0 \in C$. If the proposal is accepted by all members of C, the game stops and the government is $\{C, \mathbf{x}\}$. If the proposal is not unanimously accepted by the parties in C, then bargaining within the coalition continues. To avoid spurious equilibria in which no party is ever pivotal, we assume that parties in C vote sequentially in some order.[12] At period $t + \Delta$ a different member i of C is recognized to make an offer to the coalition: again the proposal is accepted by all members of C, the game stops; otherwise the process continues. Payoffs after an offer is rejected are discounted with discount factor $\beta_\Delta < 1$.

In the baseline model, we assume that parties in C are ordered according to some permutation $\iota(k, C)$, so the proposer following at the kth stage for $k \leq n(C)$ is identified as $\iota(k, C)$. The formateur is always the first proposer ($\iota(1, C) = f$) and the order is periodic ($\iota(n(C) + i, C) = \iota(i, C)$), so after all members of the coalition have a chance to make a proposal, proposal power returns to the formateur. Every time that the formateur is recognized again, s/he can continue with the same coalition as in the previous rounds or move to a different coalition C'. This reflects the fact that the formateur is not bound to a specific coalition and thus can strategically choose to change "partners."

At any point in time after which a proposal is rejected and before a new proposal is made we have a negotiation breakdown with probability p_Δ. In case the negotiation is interrupted,

[12] If, for example, all parties are expected to vote no in a simultaneous vote, then voting no is always optimal since no party is pivotal. An alternative solution to this problem, common in voting games, is to assume the mild refinement requiring that parties vote as if they were pivotal.

formateur f loses the status of formateur and a new formateur in $\mathcal{T}$ is appointed. If bargaining breaks down when the formateur is f^t with $t < T$, formateur f^{t+1} is selected according to an exogenous acyclical rule $f^{t+1} = \Psi(f^t)$. Allowing for $p < 1$ generalizes the assumption in the existing literature that breakdowns occur with probability one after the formateur's offer is rejected and allows bargaining within the coalition to go beyond a simple "take-it-or-leave-it" offer. We assume that β_Δ and p_Δ are continuous and differentiable functions of Δ with bounded derivatives and, respectively, decreasing in Δ and converging to 1 as $\Delta \to 0$, and increasing and converging to 0 as $\Delta \to 0$. We moreover assume that the limit of the ratio $(1 - \beta_\Delta)/p_\Delta$ as $\Delta \to 0$ exists and equal to θ (which may possibly be zero or infinite). In the following, whenever we take Δ as given and it does not generate confusion, we will simply refer to β_Δ and p_Δ as β and p.

In Section 3 we first assume that if the last formateur f^T fails to form a government, new elections are held, yielding utilities $\mathbf{u} = (u_1, ..., u_n)$. In Section 4 we endogenize the reservation utilities assuming that in case f^T fails to form a government, the process repeats itself restarting from f^0, so that $f^0 = \Psi(f^T)$. The case with exogenous outside options corresponds to a case in which failure to form the government leads to a new regime: for example a caretaker government or new elections with an exogenously given outcome. The case in which the process restarts with f^0 corresponds to a situation in which even if there are new elections, the bargaining positions in congress are not expected to change in a significant way, thus leading to the same strategic situation at the government formation stage. When the outside option is exogenous, we assume that reaching an agreement is better than obtaining $\mathbf{u}$, formally $V(C) \geq \sum_{i \in N} u_i$ for at least a $C \in \mathcal{C}$. Given this, a coalition C is efficient if and only if $V(C) = \max_{C' \in \mathcal{C}} V(C')$.

An history h^t is defined in the usual way, as a description of the sequence of formateur and, for each of them, the sequence of coalition selections, proposals and votes. A (pure) strategy for a player i assigns a choice of coalition $C(h^t)$ and a vector of proposals $\mathbf{x}_i = \{x_{i,1}(h^t), ..., x_{i,n(C(h^t))}(h^t)\}$ in $X(C(h^t))$ to each history h^t at which the player is formateur and proposer; a vector of proposals $\mathbf{x}_i = \{x_{i,1}(h^t), ..., x_{i,n(C(h^t))}(h^t)\} \in X(C(h^t))$ to each history h^t at which $C(h^t)$ is the coalition and i is proposer but not formateur; and an acceptance thresholds $a_i(h^t)$ to each history h^t at which the player is a responder. A (pure) stationary strategy for a player i assigns a choice of coalition (C_i) and proposal $\mathbf{x}_i = \{x_{i,1}(C_i), ..., x_{i,n(C_i)}(C_i)\} \in X(C_i)$ when i is formateur and proposer; a vector of proposals $\mathbf{x}_i = \{x_{i,1}((C,j)), ..., x_{i,n(C)}((C,j))\} \in X(C)$ when the coalition is C and j is the formateur; and an acceptance threshold $a_{l,i}(C,j)$ when the coalition is C, l is the proposer, i is the responder and j is the formateur. A stationary equilibrium is a Subgame Perfect Equilibrium in stationary strategies. Following a standard assumption in the literature, in the following we focus on stationary equilibria in pure strategies and for simplicity we refer to them simply as equilibria.

In the following we are interested in the welfare properties of the equilibria. We say that an equilibrium is *efficient* if the efficient coalition is selected in equilibrium with probability one. In an efficient equilibrium we might have that the first formateur who manages to form a government chooses an efficient coalition, but some other formateur forms an inefficient coalition out of the equilibrium path. In this case the welfare properties depend on the exact identity of the first formateur. We say that an equilibrium is *strongly efficient*, if the efficient coalition is selected in equilibrium with probability one starting from any possible formateur.

3 Bargaining with exogenous outside options

To study the equilibrium with finite rounds of formateurs and exogenous outside options, we start from the simple case in which there is only one formateur, i.e. $|\mathcal{T}| = 1$: in this case if bargaining breaks down, parties receive their outside options $\mathbf{u}$. The solution of this case will be key for the characterization of the more complicated cases and it will facilitate understanding of the logic behind the equilibria.

Assume an equilibrium exists and, in this equilibrium, a coalition C_f is chosen by formateur f with payoffs equal to $\mathbf{x}_f^* = \{x_{f,1}^*, ..., x_{f,n(C_f)}^*\}$ with $\mathbf{x}_f^* \in X(C_f)$. We can now characterize the payoffs that would be achieved if f decides to choose a generic coalition C (that may or may not coincide with C_f). The acceptance threshold of a player i in C at stage $n(C)$ of bargaining is:

$$a_{\iota(n(C),C),i}(C, C_f) = \beta \left[p u_i + (1-p) x_{f,i}^* \right] \tag{1}$$

where we are using here the notation $a_{j,i}(C, C_f)$ to indicate the acceptance threshold of i when j proposes in coalition C and the expected equilibrium coalition is C_f to emphasize that the threshold depends on C_f. Player i knows that if s/he refuses the offer one of two events occurs: with probability p, there is a bargaining breakdown, in which case the utility is u_i; with probability $1-p$, formateur f is recognized after the $n(C)$th proposer. At this stage, if C is equal to C_f, then the game continues recursively; if instead C is different than C_f, then the formateur is expected to return to C_f. This implies that party i expects to receive $x_{f,i}^*$. At this stage we can also easily define the proposal by proposer $\iota(n(C), C)$ as follows. If

$$V(C) - \sum_{i \in C \setminus \iota(n(C),C)} a_{\iota(n(C),C),i}(C, C_f) < a_{\iota(n(C),C),\iota(n(C),C)}(C, C_f), \tag{2}$$

then $\iota(n(C), C)$ cannot make a proposal that is acceptable and that guarantees him/her the reservation utility, so the proposal at the $n(C)$ stage fails and the formateur is recognized again as proposer if there is not a bargaining breakdown. In this case the expected discounted payoff at the beginning of stage $n(C)$ is $\beta \left[p u_i + (1-p) x_{f,i}^* \right]$ for all players. If (2) is not sat-

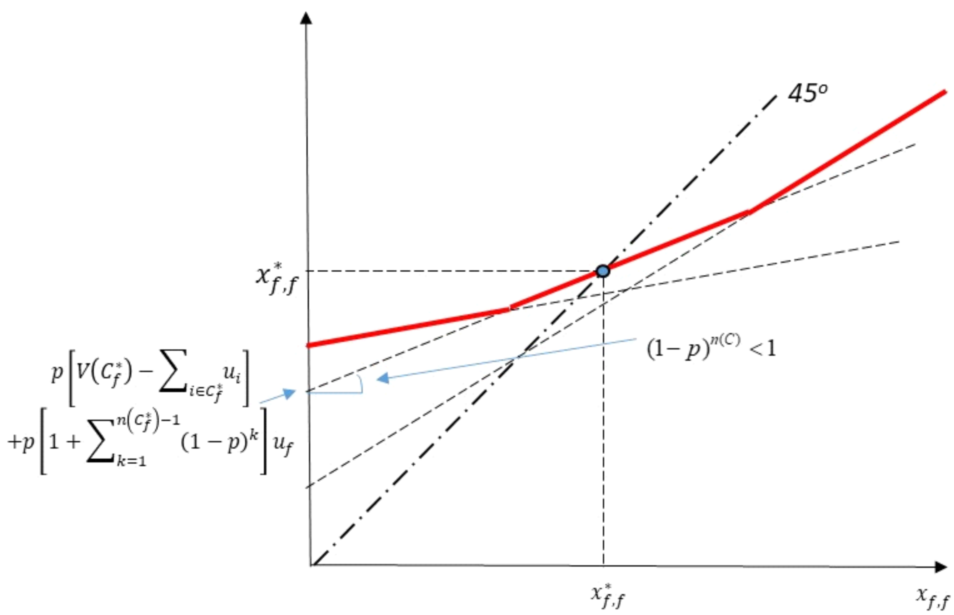

Figure 1: The selection of the equilibrium coalition.

isfied, instead, we have $x_{\iota(n(C),C),i}(C, C_f) = a_{\iota(n(C),C),i}(C, C_f)$ for all $i \in C \backslash \iota(n(C), C)$ and $x_{\iota(n(C),C),\iota(n(C),C)}(C, C_f) = V(C) - \sum_{i \in C \backslash \iota(n(C),C)} a_{\iota(n(C),C),i}(C, C_f)$.

Proceeding in the same way by backward induction, we can uniquely define the acceptance threshold for all bargaining stages up to the first, when the formateur makes a proposal for the first time. At this stage, f proposes $a_{f,i}(C, C_f)$ to all other $i \in C \backslash f$, securing a payoff $V(C) - \sum_{i \in C \backslash f} a_{f,i}(C, C_f)$ if this is larger than $a_{f,f}(C, C_f)$, or a payoff $a_{f,f}(C, C_f)$ otherwise. The initial coalition C_f is chosen in equilibrium if and only if it is a fixed-point of the following correspondence that maps coalitions to coalitions:

$$C_f \in \arg\max_{C \in \mathcal{C}_f} \left\{ V(C) - \sum_{i \in C \backslash f} a_{f,i}(C, C_f) \right\}. \tag{3}$$

When C_f satisfies (3), then it is indeed optimal for f to select it whenever s/he has proposal power. In this case, therefore, the expectation that after stage $n(C)$ the payoff will be $\mathbf{x}^*_f$ is correct. The following result tells us that a fixed-point of (3) exists and it is generically unique. Let

$$C^*_f = \arg\max_{C \in \mathcal{C}_f} \left\{ \frac{1 - \beta(1-p)}{1 - [\beta(1-p)]^{n(C)}} \left[V(C) - \frac{\beta p}{1 - \beta(1-p)} \sum_{l \in C} u_l \right] \right\}. \tag{4}$$

Naturally, since the set of coalitions is finite, C^*_f is well defined and, except for a non-generic choice of payoffs, unique. We have:

Lemma 1. *For a generic choice of payoffs, C_f^* is the unique fixed-point of (3).*

The idea behind Lemma 1 can be easily illustrated. In the appendix we explicitly solve for the acceptance thresholds $a_{\iota(\tau,C),i}(C;C_f)$ for all stages $\tau = 1,..,n(C)$ and all $i \in C$ and show that the equilibrium payoff obtained by f can be represented in a recursive way as:

$$x_{f,f}^*(C_f, C_f) = \max_{C \in \mathcal{C}_f} \left\{ \begin{array}{c} [1-\beta(1-p)] \left[\begin{array}{c} V(C) \\ -\frac{\beta p}{1-\beta(1-p)} \sum_{l \in C} u_l \end{array} \right] \\ +\beta p \left[1 + \sum_{k=1}^{n(C)-1} (1-p)^k \beta^k \right] \cdot u_f + [\beta(1-p)]^{n(C)} x_{f,f}^*(C_f, C_f) \end{array} \right\}. \quad (5)$$

From (5) it can be seen that the choice of C has two effects on $x_{f,f}^*$. Assume for simplicity, and only for the sake of this discussion, that the reservation utilities $(u_i)_{i \in N}$ are zero. On the one hand, the choice of C affects the efficiency of the allocation, as represented by the intercept of (5) with the vertical axis in Figure 1, which is proportional to $V(C)$ when $u_i = 0$ for all i. Naturally the formateur would like to choose an efficient coalition, since this guarantees a larger pie to be divided. On the other hand, the choice of C affects the fraction of surplus that can be extracted by f: specifically, the larger is the coalition, the lower is the bargaining power of f. This is the hold-up problem and it is reflected in the slope of (5), that is decreasing in the size of the coalition (see Figure 1). Naturally the formateur's ability to extract surplus depends also on the probability of breakdown. When the probability is high, the formateur's offer is basically a take-it-or-leave-it offer, and all surplus can be extracted: in this case the lines are almost flat; and C_f^* converges to the surplus maximizing coalition. As p decreases, however, the formateur's commitment power and the share of surplus that s/he can appropriate is reduced.

Note that for all Cs, the term in the brackets in (5) is a contraction. It follows that the upper contour is a contraction as well, as illustrated by the thick line in Figure 1. We must therefore have a unique fixed-point $x_{f,f}^*$ and associated to it a (generically) unique optimal coalition for f that balances efficiency with bargaining power.

Given Lemma 1, we can now easily show that generically there is a unique equilibrium and it must be supported by parties in C_f^*. By construction, every time that the formateur has an opportunity of choosing C, s/he will chose C_f^*. It follows that in equilibrium C_f^* is chosen and we must satisfy for all $j \in C_f^*$

$$\begin{aligned} a_{i,j}^* &= \beta \left[p u_j + (1-p) x_{\iota(\iota^{-1}(i,C)+1,C),j}^* \right] \text{ for all } i \in C_f^* \\ x_{j,j}^* &= V(C_f^*) - \sum_{i \in \mathcal{C}_f^* \backslash j} a_{j,i}^* \text{ and } x_{j,i}^* = a_{j,i}^* \text{ for all } i \in \mathcal{C}_f^* \backslash j. \end{aligned}$$

In the appendix we show that this system is reduced to a system of $n(C_f^*) \times n(C_f^*)$ equations in $n(C_f^*) \times n(C_f^*)$ unknowns that has a unique solution. Using this solution we obtain the equilibrium payoffs as $x^*_{f,i}$ for all $i \in C^*_f$ and u_j for $j \in N \backslash C^*_f$; and the equilibrium strategies when C^*_f is chosen. Using these equilibrium values we can also uniquely define the strategies in all out of equilibrium subgames in which some other coalition C are chosen. Define the payoffs:

$$\begin{aligned} x^*_f(C, \mathbf{u}) &= \frac{\beta p}{1-\beta(1-p)} u_f + \frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C^*_f)}} \left[V(C^*_f) - \frac{\beta p}{1-\beta(1-p)} \sum_{l \in C^*_f} u_l \right] \qquad (6) \\ x^*_i(C, \mathbf{u}) &= \frac{\beta p}{1-\beta(1-p)} u_i + \frac{[\beta(1-p)]^{\iota^{-1}(i, C^*_f)-1}[1-\beta(1-p)]}{1-[\beta(1-p)]^{n(C^*_f)}} \left[\begin{array}{c} V(C^*_f) \\ -\frac{\beta p}{1-\beta(1-p)} \sum_{l \in C^*_f} u_l \end{array} \right] \text{ for } i \neq f \end{aligned}$$

We therefore have:[13]

Proposition 1. *The bargaining game has a unique stationary equilibrium in which coalition C^*_f defined in (4) is selected and the payoffs are uniquely defined as $x^*_f\left(C^*_f, \mathbf{u}\right)$ for the formateur and $x^*_i\left(C^*_f, \mathbf{u}\right)$ for party $i \in C^*_f \backslash f$ who is in position $\iota^{-1}(i, C^*_f)$ in the bargaining queue. All other parties in $N \backslash C^*_f$ receive zero ($x^*_l\left(C^*_f, \mathbf{u}\right) = 0$ for $l \in N \backslash C^*_f$).*

To gain insight on the equilibrium allocation, it is useful to consider the case in which the interaction between the parties is very frequent, that is when $\Delta \to 0$. This is important for two reasons. First, because it will give us a simple characterization of the payoffs that will be useful in the generalization studied in the next section. Second, because it highlights an interesting connection between the model of the previous section and the *Nash Bargaining Solution* (henceforth NBS). In the special case in which $n = 2$, the bargaining game considered in the previous section coincides with Rubinstein's model with the risk of breakdown (see Binmore, Rubinstein and Wolinsky [1986]). It is well known that in this case the solution of Rubinstein's model coincides with the NBS. While the NBS formula can be extended to n players, there are two basic reasons why a mechanical extension is not advisable. First, because there is not a unique way to rationalize the n-person NBS as the limit of a noncooperative game. Secondly, because even if one were to accept the specific bargaining protocol that rationalizes the n-person bargaining solution, then the solution would imply that the coalition of all players is formed and all players share the surplus. In the political context studied in this paper, such a scenario is highly unlikely. Ideally, a n-person generalization of NBS would provide indication of *which* coalition is selected and *how* surplus is divided in that coalition.

To analyze the limit as $\Delta \to 0$ we need to specify the rate at which β_Δ converges to 1 relative to the rate at which p_Δ converges to zero. When $(1-\beta_\Delta)/p_\Delta \to \infty$ it is as if as

[13] A complete characterization of the equilibrium strategies is presented in the Proof of Proposition 1 in the appendix.

the frequency of interactions increases, the parties become increasingly more concerned about the delayed gratification implied by their time preferences, and less about the probability of a change in formateur and its strategic implications. When $(1-\beta_\Delta)/p_\Delta \to 0$, the opposite is true, implying that the parties are increasingly concerned about the probability of a bargaining breakdown as $\Delta \to 0$, rather than on the delayed gratification. It seems reasonable to assume that, as the frequency of interactions increases, the parties become more concerned about the probability of a breakdown and of a change of formateur, rather than on the delayed gratification: in a negotiation that may last days or at most weeks, the parties are more concerned about the strategic implications of a change in formateur than on the fact that their share of surplus may be delayed by a few days. The assumption that $(1-\beta_\Delta)/p_\Delta \to 0$ is also the assumption made by Binmore et al. [1985] in their two person bargaining model, where indeed it is assumed that $\beta_\Delta = 1$ for any Δ. In the following, we assume $(1-\beta_\Delta)/p_\Delta \to 0$ as our leading case; we will however discuss the implication of alternative assumption as we present the results in the following sections.

Assumption 1. *As* $\Delta \to 0$, $(1-\beta_\Delta)/p_\Delta \to 0$.

The following result shows that under Assumption 1 the equilibrium in Proposition 1 provides an alternative generalization of the Nash solution to the n-person case as the bargaining interval converges to zero. Define the *C-Nash Bargaining Solution* as:

$$\mathcal{N}(C,\mathbf{u}) = \arg\max_{\mathbf{x}\in X(C)} \prod\nolimits_{l\in C} \left[x_l - u_l\right]. \tag{7}$$

where $\mathcal{N}(C,\mathbf{u}) = \{\mathcal{N}_1(C,\mathbf{u}), ..., \mathcal{N}_{n(C)}(C,\mathbf{u})\}$. This is the Nash bargaining solution when coalition C is chosen. Let, moreover, $\overline{C}_f$ be the coalition with the largest per capita surplus such that $f \in \overline{C}_f$, i.e. that maximizes $\left[V(C) - \sum_{l\in C} u_l\right]/n(C)$ for $C \in \mathcal{C}_f$ and extend $\mathcal{N}(C,\mathbf{u})$ to the players outside C by setting $\mathcal{N}_j(C,\mathbf{u}) = 0$ for $j \in N \backslash C$. We have:

Proposition 2. *Under Assumption 1, the equilibrium of the bargaining problem converges to* $x^* = \mathcal{N}(\overline{C}_f, \mathbf{u})$ *as* $\Delta \to 0$.

Proposition 2 can be seen as a generalization of Binmore, Rubinstein and Wolinsky [1986], who have provided the first non-cooperative microfoundation of the Nash bargaining solution with 2 players. What sets this result apart from the 2-player case and other microfoundations with n players is that in Proposition 2 the equilibrium coalition is endogenous, the environment does not require superadditivity, and the equilibrium is unique in the class of stationary equilibria.[14]

[14] For games in which all the coalitions have the same value, specific bargaining protocols achieving the Nash Bargaining Solution with the grand coalition N are presented by Chae and Yang [1994] and Krishna and Serrano [1996], among others. The protocols used in these papers are different than that of Baron and Ferejohn [1989]

If we abandon Assumption 1, the analysis remains qualitatively the same if we assume that $(1-\beta_\Delta)/p_\Delta \to \theta$, for some finite θ. In this case, the equilibrium converges to $\mathcal{N}(C_\xi, \xi\mathbf{u})$ where $\xi = (1+\theta)^{-1} < 1$ and C_ξ is the coalition that maximizes $\left[V(C) - \sum_{l\in C}(\xi u_l)\right]/n(C)$. Essentially, the equilibrium corresponds to a Nash bargaining solution in which the outside options are weighted by ξ. Even when we endogenize the reservation utilities $\mathbf{u}$, the analysis is qualitatively the same as in the previous analysis with the only difference that now we have an additional parameter. We will discuss this more in the following sections.

The analysis becomes significantly different if $(1-\beta_\Delta)/p_\Delta \to \infty$ as $\Delta \to 0$. In this case, the parties are unconcerned by the possibility of a bargaining breakdown as $\Delta \to 0$, they are exclusively concerned by the delay in gratification. The equilibrium converges to the equilibrium we would have without bargaining breakdown, so it is independent of the reservation utilities. The equilibrium coalition is the C' that maximizes $V(C)/n(C)$ for $C \in \mathcal{C}_f$, and we have $x_i^* = V(C')/n(C')$ for $i \in C'$ and $x_i^* = 0$ for the others. We should note that the same would have happened in Binmore et al. [1985] if they allowed for a discount factor $\beta_\Delta < 1$ and $(1-\beta_\Delta)/p_\Delta \to \infty$ as $\Delta \to 0$.

4 Endogenizing the outside options

In the previous section we have assumed that if bargaining with the formateur fails, then a caretaker is appointed or there are new elections and the parties receive exogenously specified expected utilities $(u_i)_{i\in N}$. It is however common in legislative processes that if there is a bargaining breakdown, then a new party is selected as formateur and the process restarts. By explicitly modelling what happens after a bargaining breakdown, we can endogenize the outside options. This is important because it allows us to explain commonly observed phenomena such as low or negative formateurs' premia, supermajorities and strategic delays, without making ad hoc assumptions on exogenous outside options.

To extend the theory of the previous section to environments with multiple formateurs, let $\mathcal{T} = \{0, ..., T\} \subseteq N \cup \{0\}$ be the set of formateurs and let us assume that after breakdown of bargaining with formateur $t \in \mathcal{T}$, formateur $t+1$ is called to the job. In a finite horizon extension after $t = T$, the players receive exogenous utilities $\mathbf{u}_{T+1} = \{u_1, .., u_n\}$ as in the previous section. In an infinite horizon extension, after $t = T$, formateur $t = 0$ is called again to the job and the process repeats. If no agreement is ever reached, the agents' payoffs are zero. We make the

and the model described above since they rely on the possibility of players to commit to partial agreements. For games in which coalitions may generate heterogeneous values, specific bargaining protocols achieving the Nash Bargaining Solution have been presented by Okada [2010], who focuses on stationary equilibria assuming that the grand coalition N forms; and Gomes [2019], who focuses on Strong Nash Stationary equilibria, in which players are assumed to be able to coordinate to the most efficient outcome.

assumption of a stationary order of formateurs since it seems a natural first step.[15]

Define $\mathcal{C}_t^*(\mathbf{u},\Delta)$ as the following family of coalitions in $\mathcal{C}_t$:

$$C_t^*(\mathbf{u},\Delta) := \arg\max_{C\in\mathcal{C}_t} \left\{ \frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C)}} \begin{bmatrix} V(C) \\ -\frac{\beta p}{1-\beta(1-p)} \sum_{l\in C} u_l \end{bmatrix} \right\}.$$

where we highlight in the definition that the set depends on Δ since p and β are both functions of Δ. If formateur t is able and willing to form a government C when the reservation utilities are $\mathbf{u}$, then we have $C \in \mathcal{C}_t^*(\mathbf{u},\Delta)$ and the payoffs are $x_i^*(C,\mathbf{u})$ as defined in Proposition 1. If the formateur is unable or unwilling to form a government, then the expected payoffs are $[\beta p/(1-\beta(1-p))]\, u_i$ for all $i \in C$.[16] Formateur t is able and willing to form a government $C \in C_t^*(\mathbf{u},\Delta)$ only if $x_i^*(C,\mathbf{u})$ is larger than the utility of waiting for all $i \in C$. A government C is therefore formed by t only if $V(C) - [\beta p/(1-\beta(1-p))] \cdot \sum_{l\in C} u_l \geq 0$. Given this, define:

$$F_i^{\Delta}(\mathbf{u},C) = \begin{cases} x_i^*(C,\mathbf{u}) & \text{if } V(C) - [\beta p/(1-\beta(1-p))] \cdot \sum_{l\in C} u_l \geq 0 \\ [\beta p/(1-\beta(1-p))]\, u_i & \textit{else} \end{cases}$$

and $F^{\Delta}(\mathbf{u},C) = \{F_1^{\Delta}(\mathbf{u},C), ..., F_n^{\Delta}(\mathbf{u},C)\}$, where we add Δ in the superscript to highlight that $x_i^*(C,\mathbf{u})$, p and β all depend on Δ. We can now define the following operator. At the last stage, when T is formateur:

$$F_T^*(\mathbf{u},\Delta) := \left\{ \mathbf{v} \text{ s.t. } \mathbf{v} = F^{\Delta}(\mathbf{u},C) \text{ for some } C \in \mathcal{C}_T^*(\mathbf{u},\Delta) \right\}.$$

The operator $F_T^*(\mathbf{u},\Delta)$ maps the expected reservation utilities $\mathbf{u}$ if there is a bargaining breakdown with the last formateur T, to the equilibrium utilities that are reached with formateur T. For the previous stages, we define the payoffs recursively as:

$$F_{t-1}^*(\mathbf{u},\Delta) := \left\{ \mathbf{v} \text{ s.t. } \mathbf{v} = F^{\Delta}(\mathbf{u}^t,C) \text{ for some } C \in \mathcal{C}_{t-1}^*(\mathbf{u}^t,\Delta) \text{ and } \mathbf{u}^t \in F_t^*(\mathbf{u},\Delta) \right\}.$$

The operator $F_{t-1}^*(\mathbf{u},\Delta)$ maps the reservation utilities $\mathbf{u}$ to the utilities reached in equilibrium if formateur $t-1$ is reached. For a generic choice of $\mathbf{u}$, $\mathcal{C}_{t-1}^*(\mathbf{u},\Delta)$ is a singleton and thus either bargaining with $t-1$ fails to form a government and we move to t, or there is a unique optimal coalition for $t-1$, thus $F_{t-1}^*(\mathbf{u},\Delta)$ is a function. It is however convenient to allow $F_{t-1}^*(\mathbf{u},\Delta)$ to be a correspondence since when $\mathbf{u}$ is endogenous, the formateur may be indifferent among different

[15] In Section 6.2 we also study the case in which the order of formateurs is random as in Baron and Ferejohn [1989].

[16] The payoff x_i of waiting for a bargaining breakdown solves $x_i = \beta p u_i + \beta(1-p)x_i$.

coalitions. The operator $F_0^*(\mathbf{u}^*, \Delta)$ maps the expected reservation utilities $\mathbf{u}$ to the equilibrium utilities that are reached with the first formateur. We define:

Definition 1. *An equilibrium outcome in pure strategies of the extended bargaining game is a vector of utilities* $\mathbf{u}^*$ *and a coalition* C^* *such that* $\mathbf{u}^* \in F_0^*(\mathbf{u}^*, \Delta)$ *and* $C^* \in \mathcal{C}_0^*(\mathbf{u}^*, \Delta)$.

We study here the infinite horizons extension as $\Delta \to 0$. We will therefore focus on equilibrium outcomes that are the limit of equilibria as $\Delta \to 0$.

The first natural question to ask is whether a strongly efficient equilibrium exists, that is an equilibrium in which all formateurs either propose the efficient coalition or are unable to form a coalition. The following result establishes an (essentially) necessary and sufficient condition for a strongly efficient equilibrium to exist as $\Delta \to 0$. Let C^{**} be the efficient coalition. We have:

Proposition 3. *Under Assumption 1, a strongly efficient equilibrium exists and is the limit of equilibria as* $\Delta \to 0$ *only if* $V(C^{**}) \geq [n(C^{**}) \backslash n(C \cap C^{**})] \cdot V(C)$ *for all* $C \in \mathcal{C} \backslash C^{**}$*, and it exists if this condition is satisfied as a strict inequality. When it exists, payoffs in the strongly efficient equilibrium are* $x_i^* = V(C^{**}) \backslash n(C^{**})$ *if* $i \in C^*$*, and* $x_i^* = 0$ *if* $i \in N \backslash C^{**}$.

The existence of a strongly efficient equilibrium requires the efficient coalition C^{**} to be significantly more efficient than any other feasible coalition: players excluded from C^{**} are cheap to "buy" since in equilibrium they receive zero; existence requires that no formateur is tempted to move to another coalition C, by convincing the subset $C \cap C^{**}$ to make the change. What is remarkable in Proposition 3 is that under Assumption 1 the necessary and sufficient condition can be stated independently of other details of the model, including the order of formateurs and the order of proposals in the intracoalitional bargaining stage in C.

There are three lessons from Proposition 3. The first and most obvious is that there is a simple condition to check for the existence of a strongly efficient equilibrium; and that this condition can be satisfied in natural examples. Consider the following two examples:

Example 1. Assume there are 3 parties, $1, 2, 3$, with order of formateurs $1 \to 2 \to 3$, who deliberate by majority rule. In this case, the supermajority $\{1, 2, 3\}$ is a strongly efficient equilibrium as $\Delta \to 0$ if $V(\{1, 2, 3\})/3 > \max_{C \in \mathcal{C} \backslash \{1,2,3\}} V(C)/2$, that is if its associated average payoff is maximal. The minimal winning coalition $\{i, j\}$, instead, is a strongly efficient equilibrium if $V(\{i, j\})/2 > \max\{V(\{i, k\}), V(\{j, k\}), V(\{1, 2, 3\})/2\}$.

Example 2. Assume there are 4 parties: a moderate leftist party, M_l; a moderate rightist party, M_r; a more extreme leftist party, L; and a more extreme rightist party, R. Suppose that the possible coalitions are only a "center left" coalition $\{M_l, M_r, L\}$, or "center right" coalition $\{M_l, M_r, R\}$. Assume, for example, that $V(\{M_l, M_r, L\}) = V_H$ and $V(\{M_l, M_r, R\}) = V_L$ with

$V_H > V_L$; and that the formateurs are $\mathcal{T} = \{M_l, M_r\}$ with order $M_l \to M_r$. In this case we have a strongly efficient equilibrium if $V_H/3 > V_L/2$. If we denote the average surplus of the efficient coalition and the alternative as a_H and a_L, a strongly efficient equilibrium exists if $a_H > (3/2) \cdot a_L$.

The second lesson is that a strongly efficient equilibrium implies no formateur's premium. This happens because any coalition needs to unanimously approve a government (else, of course, it would be a different coalition, with a different expected surplus) and the opportunity cost of a bargaining breakdown is the same for all coalition members: the time required until one of them is reappointed as formateur. As the identity of the formateur changes, this cost may also change since the number of parties outside C^{**} who are given a chance to form the government after a breakdown may change: but it is always the same for all parties in the coalition.

The third and perhaps most important lesson, however, is that if the core is empty, then there is no hope to have a strongly efficient equilibrium.[17] Since $n(C^{**}) \backslash n(C \cap C^{**}) \geq 1$, in a strongly efficient equilibrium we must have $\sum_{i \in C^{**}} x_i^* \geq V(C)$ for all $C \in \mathcal{C} \backslash C^{**}$, this is impossible with an empty core. In many environments of interest the assumption that the core is not empty is very demanding. Bargaining problems often include important redistributive components: when the redistributive motives are sufficiently important, the core is typically empty.[18] In these scenarios, we cannot hope for a strongly efficient equilibrium, still efficient equilibria may exist for specific orders of the formateurs.

To study the remaining equilibria of the extended game we now specialize the model assuming that there are three parties and each party has a chance to be formateur, so $N = \{1, 2, 3\} = \mathcal{T}$. The assumption of three parties is an assumption that has been previously adopted by Austen-Smith and Banks [1988], Baron [1991], Baron and Diermeier [2001] and many others: it allows us to keep the key strategic feature of the problem, minimizing the analytical complications. It is also realistic since many political systems have this feature.

Without loss of generality we can assume that parties are formateurs in the order of their index and the value of the coalitions are $V(\{1,2\}) = a - d$, $V(\{1,3\}) = a + e$ and $V(\{2,3\}) = a$, where $a > 0$; either d and e are nonnegative, or d and e are non-positive; and, finally, $a - d \geq 0$ and $a + e \geq 0$.[19] We also assume here that the coalition of all players $N = \{1, 2, 3\}$ does not

[17] A feasible point $\mathbf{x} = (x_1, ..., x_n)$ is in the core if it cannot be challenged by some coalition S: that is, there is no coalition of parties $S \subseteq N$ that can be formed and reward all of its members $i \in S$ with a payoff $y_i > x_i$. Formally, the *core* of a coalition structure (N, V) is defined as $\mathcal{K}(N, V) = \{x \in \mathcal{F}(N, V) \text{ s.t. } \sum_{i \in S} x_i \geq V(S)$ for any $S \subseteq N\}$, where $\mathcal{F}(N, V)$ is the set of feasible allocations, i.e. $\mathcal{F}(N, V) = \{x \in \mathcal{R}^n \text{ s.t. } x_i \geq 0$ for $i \in N$ and $\sum_{i \in N} x_i \leq \max_C V(C)\}$.

[18] For example, this is always the case when for any two $C, C' \in C$, $|V(C) - V(C')| \leq \eta$ for some sufficiently small η.

[19] To see that there is no loss of generality in this specification, note that, for example, the game with $(d, e) = (-f, -g) > 0$ and order of proposal is $1, 3, 2$ is equivalent to the game with order $1, 2, 3$ and $(d, e) = (g, f)$. The

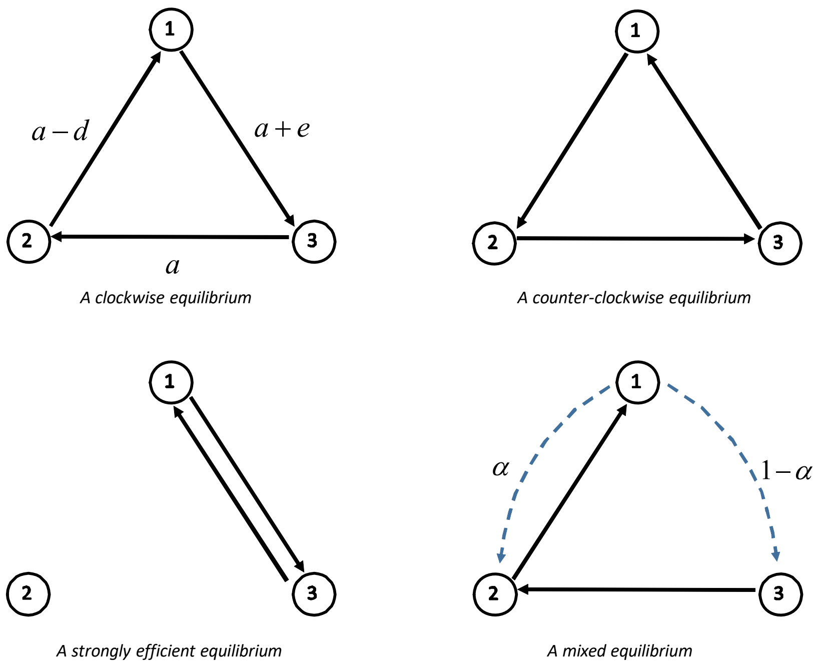

Figure 2: Classification of the possible equilibria when $d, e > 0$.

find policy agreements easier to implement than the best of the minimal winning coalitions, thus $V(\{1,2,3\}) \leq \max_{C\in\mathcal{M}} V(C)$.[20] Under this condition, as it can be easily verified, $\{1,2,3\}$ is never the equilibrium coalition, and we can focus on minimal winning coalitions. We relax this assumption in Section 5 where we study supermajorities.

Our first result characterizes the strategies that are feasible in a pure strategy equilibrium. Strategies that are not strongly efficient come in different types and shapes. Two forms of inefficient strategies are particularly salient.

Definition 2. *A clockwise equilibrium is an equilibrium in which, when given a chance, party* 1 *selects* 3*, party* 3 *selects* 2 *and party* 2 *selects* 1.

Definition 3. *A counterclockwise equilibrium is an equilibrium in which, when given a chance, party* 1 *selects* 2*, party* 2 *selects* 3 *and party* 3 *selects* 1.

The top left panel of Figure 2 illustrates the clockwise equilibrium, where the choice of coalition

game with, say, $d > 0$ and $e < 0$ with $|d| \leq |e|$ and order $1 \to 2 \to 3$ is equivalent to a game with the same order and payoffs: $V(\{1,3\}) = a' - d'$, $V(\{1,2\} = a'$ and $V(\{2,3\}) = a' + e'$ with $a' = a - d$, $d' = -(e+d) > 0$ and $e' = d > 0$. Moreover, since the Markov equilibrium is memoryless, once we have characterized the equilibrium with some order starting from 1, we have also characterized any game with the same order starting from 2 or 3.

[20] The idea is that if a winning coalition can agree on a policy that generates a surplus $V(C)$, then additional member who can veto it in a larger coalition C' can only reduce the attainable value.

is illustrated by a pointed arrow (from the chooser to the chosen). The top right and bottom left panels illustrate the counterclockwise and strongly efficient equilibria with $d, e \geq 0$ respectively[21] . This classification is important because, as the following result shows, it exhausts all the possible cases that can occur in equilibrium. In the appendix we prove that:

Proposition 4. *Under Assumption 1, a pure strategy equilibrium is either clockwise, counterclockwise or strongly efficient.*

The intuition behind Proposition 4 is simple. Assume, for the sake of the discussion here, that $d, e \geq 0$. Suppose that 1 chooses 2 to be in a coalition with her.[22] Is it possible that 3 also finds it optimal to select 2 as her coalition partner if given a chance to be formateur? The problem with such a scenario is that if this were the case, then 2 would always be in a coalition, thus making her equilibrium outside option high (and making the equilibrium outside option of 3 very low); this would reduce the incentive to include 2 in a coalition, and make 3 more appealing. But then it is natural to expect that 1 will choose 3, since a coalition $\{1, 3\}$ is more valuable and 3 is "cheap." Proposition 4 shows that indeed 3 will not find it optimal to include 2, thus leaving only two other scenarios: either offers are "spread out" following the order of the formateurs (the counterclockwise equilibrium); or offers are spread out following the opposite order (the clockwise equilibrium).

We can now turn to the characterization of the equilibria. Note that the game can be seen as a classic stochastic game in which the only state variable at any point in time is the identity of the formateur. Proposal strategies are easily represented by functions x_j^i, describing the proposal that i makes to j when i is the formateur.[23] Proposition 3 immediately gives us when the strongly efficient equilibrium exists: $e > a$ for $d, e \geq 0$; and $d < -a$ for $d, e < 0$. We can therefore focus on characterizing when the clockwise and counterclockwise equilibria exist.

Assume we are in a counterclockwise equilibrium and consider the problem of formateur 1. She selects party 2 and makes a payment that depends on 2's expected reservation utility, that is the utility that 2 expects to achieve if bargaining with 1 breaks down. The reservation utilities at this stage correspond to x_j^2, the payment that formateur 2 in equilibrium offers to j. From

[21] The bottom right panel illustrates a mixed equilibrium that will be discussed below.

[22] As we will see, this may occur in equilibrium even if $d, e > 0$.

[23] We adopt the simplified notation here since it will not lead to confusion. In terms of the more general notation of Section 2, we have $x_j^i = x_j(\{i, j\}, i)$.

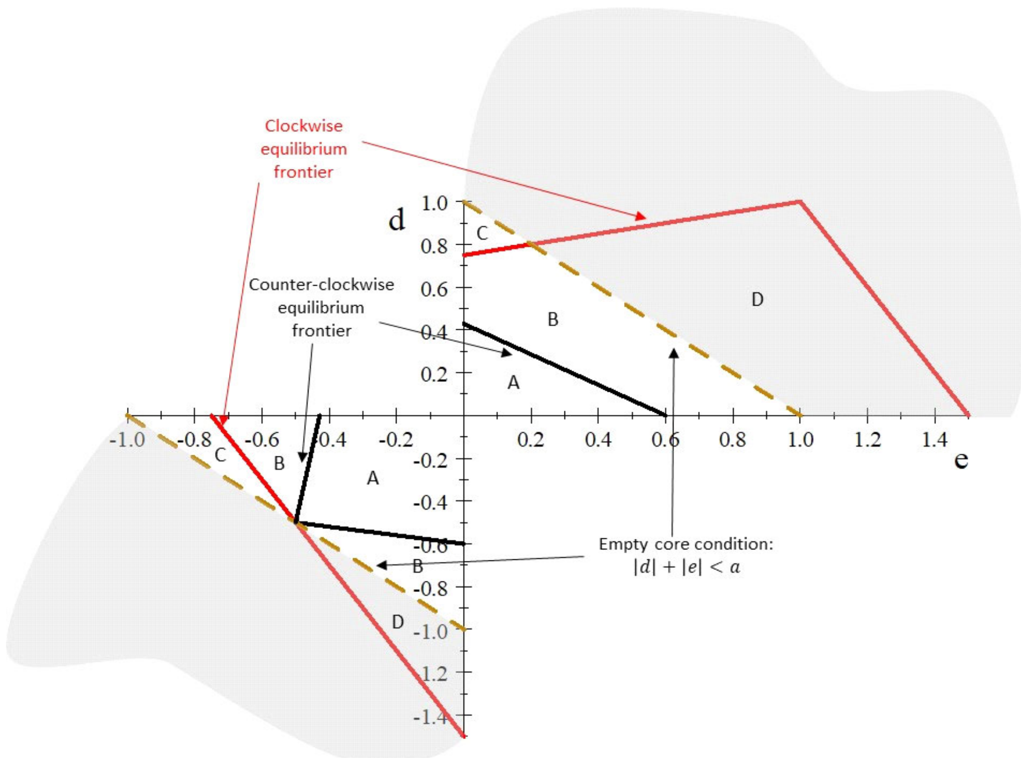

Figure 3: The equilibrium characterization with an empty core for $a = 1$.

Proposition 2, x_j^1 must satisfy as $\Delta \to 0$:[24]

$$x_1^1 = x_1^2 + \frac{a - d - x_1^2 - x_2^2}{2},\ x_2^1 = x_2^2 + \frac{a - d - x_1^2 - x_2^2}{2},\ x_3^1 = 0. \tag{8}$$

Compared to the analysis of Section 3, now the parties' reservation utilities are endogenous and they themselves depend on what is expected to happen if bargaining with formateur 2 breaks down. Following the same logic as in (8), we obtain the allocations when 2 and 3 are formateurs:

$$\begin{aligned} x_1^2 &= 0,\ x_2^2 = \frac{a}{2} - \frac{x_3^3}{2},\ x_3^2 = \frac{a}{2} + \frac{x_3^3}{2} \\ x_1^3 &= \frac{a+e}{2} + \frac{x_1^1}{2},\ x_2^3 = 0,\ x_3^3 = \frac{a+e}{2} - \frac{x_1^1}{2}, \end{aligned} \tag{9}$$

[24] The formulas look analogous, just a little more complicated when $\Delta > 0$. For example, we would have: $x_1^1 = \xi_\Delta x_1^2 + \phi_\Delta^1 \left[a - d - \xi_\Delta \left(x_1^2 + x_2^2\right)\right]$, $x_3^1 = 0$, $x_2^1 = \xi_\Delta x_2^2 + \phi_\Delta^2 \left[a - d - \xi_\Delta \left(x_1^2 + x_2^2\right)\right]$, where $\xi_\Delta = \beta_\Delta p_\Delta / \left[1 - \beta_\Delta(1 - p_\Delta)\right]$. and $\phi_\Delta^1 = \frac{1-\beta_\Delta(1-p_\Delta)}{1-[\beta_\Delta(1-p_\Delta)]^2}$ and $\phi_\Delta^2 = p(1-p)\phi_\Delta^1$, with $\xi_\Delta \to 1$ and ϕ_Δ^1,$\phi_\Delta^1 \to 1/2$ as $\Delta \to 0$. Analogous formula can be derived for the reservation values in (9) below.

Equations (8)-(9) define a system of nine equations in nine unknowns that gives us the following unique solution:

$$\begin{aligned} x_1^1 &= \frac{3a-4d+e}{9},\ x_2^1 = \frac{6a-5d-e}{9},\ x_3^1 = 0 \\ x_1^2 &= 0,\ x_2^2 = \frac{3a-d-2e}{9},\ x_3^2 = \frac{6a+d+2e}{9} \\ x_1^3 &= \frac{6a-2d+5e}{9},\ x_2^3 = 0,\ x_3^3 = \frac{3a+2d+4e}{9}. \end{aligned} \tag{10}$$

These are the equilibrium allocations under the assumption that we are in a counterclockwise equilibrium. To complete the characterization we need to make sure that the coalitions chosen in this type of equilibrium are the players' best responses. Let $S_t(C)$ be the average surplus in coalition C when t is the formateur, that is:

$$S_t(C) = \frac{1}{2}\left[V(C) - (\textstyle\sum_{j\in C} x_j^{t+1})\right].$$

For illustration, let us assume here that d and e are negative, so that the most efficient coalition is $\{1,2\}$ (the case in which d and e are positive is very similar and presented in the appendix). Consider first the case in which 2 is the formateur. From Proposition 2, formateur 2 selects coalition $\{2,3\}$ if $S_2(\{2,3\}) \geq S_2(\{1,2\})$. From (10), we have:

$$\begin{aligned} S_2(\{1,2\}) &= \frac{1}{2}\left(a - d - x_1^3\right) = \frac{3a-7d-5e}{18} \\ S_2(\{2,3\}) &= \frac{1}{2}\left(a - x_3^3\right) = \frac{6a-2d-4e}{18}, \end{aligned}$$

so $S_2(\{2,3\}) \geq S_2(\{1,2\})$ if $d \geq -\frac{3}{5}a - \frac{1}{5}e$. Proceeding analogously we can verify that formateur 1 and 3 finds it optimal to select, respectively, coalitions $\{1,2\}$ and $\{1,3\}$ if and only if $d \leq 3a+7e$. We have a counterclockwise equilibrium when $d \geq -\frac{3}{5}a - \frac{1}{5}e$ and $d \leq 3a+7e$ (the region labelled "A" in the negative orthant of Figure 3). In the interior of these intervals the payoffs are uniquely defined and the equilibrium is strict, so it exists for $\Delta > 0$ sufficiently small. Using a similar logic we can characterize all the other feasible equilibria and obtain the following full characterization of equilibrium bargaining:

Proposition 5. *Under Assumption 1, we have that:*

- *A clock-wise equilibrium exists and is the limit of equilibria as* $\Delta \to 0$ *only if* $d \leq \min\left(\frac{3}{4}a + \frac{e}{4}, 3a - 2e\right)$ *when* $d, e > 0$ *and if* $d \geq -\frac{3}{2}a - 2e$ *when* $d, e \leq 0$*; and it exists and is the limit of equilibria as* $\Delta \to 0$ *if these conditions are strict inequalities.*

- *A counterclockwise equilibrium exists and is the limit of equilibria as* $\Delta \to 0$ *only if* $d \leq \frac{3}{7}a - \frac{5}{7}e$ *when* $d, e > 0$*, and if* $d \geq -\frac{3}{5}a - \frac{1}{5}e$ *and* $d \leq 3a + 7e$ *when* $d, e \leq 0$*; and it exists and is the limit of equilibria as* $\Delta \to 0$ *if these conditions are strict inequalities.*

The equilibrium structure is described in Figure 3.[25] Consider first the case in which the core is empty, which is the most relevant in environments such as legislative bargaining, where there is an important redistributive component. The condition for an empty core is $a > |d| + |e|$, that is the area below the dashed line in the positive orthant, and above the dashed line in the negative orthant. With an empty core the strongly efficient equilibrium cannot exist by Proposition 3, so we are left with the clockwise and counterclockwise equilibria only. The clockwise equilibrium exists in the regions labelled with the letters "A" and "B"; the counterclockwise equilibrium exists only in the regions labelled with the letter "A"; there is no pure strategy equilibrium in the regions labelled with the letter "C".[26] When the core is not empty, we know from Proposition 3 that the strongly efficient equilibrium exists if $e > a$ when $d, e > 0$, and when $d < -a$ when $d, e < 0$. Proposition 5 shows that now we can also have a clockwise equilibrium in the area labelled with the letter "D." In the remaining region, we can only have mixed strategy equilibria. Putting the two cases with and without an empty core together, Proposition 5 shows that pure strategy equilibria always exist if the heterogeneity of the coalitions is not too extreme.[27]

This characterization of the pure strategy equilibria presented above highlights three qualitative features of legislative bargaining that make the analysis distinctive from previous work: the possibility of inefficient equilibria; the possibility of multiple equilibria with different welfare properties depending on the parties' expectations of likely coalitions; and the possible lack of existence of a pure strategy equilibrium.

The issue of inefficient equilibria has been studied little in the previous literature on legislative bargaining because it focused on distributive politics. Noncooperative models a' la Baron and Ferejohn [1989] assume that all coalitions generate the same surplus, thus restricting the analysis to how surplus is allocated and making the choice of coalition irrelevant; cooperative models of bargaining (such as the Shapley value, for example), on the contrary, effectively assume that the largest coalition is the most efficient and always selected.[28] In the model studied above, instead, the focus is on which equilibrium coalition is chosen and how this choice depends on the associated surplus. Proposition 5 shows that indeed inefficient coalitions can be selected even if the formateur is a member of the efficient coalition, and the selected coalition may not even be the one that maximizes average surplus. This happens when the equilibrium "expectation" of

[25] In Figure 3, a is normalized at $a = 1$.

[26] When the core is not empty, the clockwise equilibrium continue to exist in the region labelled "D," but the counterclockwise equilibrium no longer exist. By Proposition 3, the stronly efficient equilibrium exists for $a < e/2$.

[27] Formally, if $d < 3/4a + e/4$ if $e < a$ when $d, e > 0$; and $d > -(3/2)\,a - 2e$ if $d > -a$ when $d, e < 0$.

[28] Welfare in legislative bargaining models has been explicitly studied in models involving public goods (for instance, Battaglini and Coate [2007], Volden and Wiseman [2007]), endogenous status quo (Dziuda and Loeper [2016]), and public debt with distortionary taxation (Battaglini and Coate [2008]).

some other member of the efficient coalition is too high, thus making convenient to choose a less efficient coalition. Surprisingly, this is not just one of the equilibrium outcomes, but under some condition the unique equilibrium outcome.[29] In strictly superadditive environments with the first rejector-becomes-proposer bargaining protocol (as in Chatterjee et al. [1993]) or in weakly superadditive with the random proposer protocol (as in Okada [1996]) efficient equilibria do not exist with an empty core. Relaxing the assumption of superadditive values and assuming the formateur protocol, we instead can have an efficient outcome with pure strategies even with an empty core, thus in environments that are relevant for legislative bargaining.[30] This may explain why formateur-type of bargaining protocol are common in Western democracies.

Consider now the issue of multiplicity. Assume here, for the sake of the discussion, $d, e > 0$. When the counterclockwise equilibrium exists, then the clockwise equilibrium exists as well. Contrary to what happens in Baron and Ferejohn [1989], this multiplicity of equilibria is payoff relevant.[31] For example, in the area in which both the clockwise and the counterclockwise equilibria coexist, the counterclockwise equilibrium is associated to the payoffs in (10); as we show in the proof of Proposition 5 in the appendix, the payoffs with a clockwise equilibrium are:

$$\begin{aligned} x_1^1 &= \frac{6a+5e-2d}{9},\ x_2^1 = 0,\ x_3^1 = \frac{3a+4e+2d}{9} \\ x_1^2 &= \frac{3a+e-4d}{9},\ x_2^2 = \frac{6a-e-5d}{9},\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = \frac{3a-2e-d}{9},\ x_3^3 = \frac{6a+2e+d}{9}. \end{aligned} \tag{11}$$

The payoff of party 1 when s/he is the formateur, for example, is much higher in the clockwise equilibrium than in the counterclockwise equilibrium. The multiplicity of equilibria capture the complexity of the strategic interaction in this model and is natural in this environment, since it reflects the fact that reservation utilities depend in a non trivial way on endogenous expectations of the future. Despite this, the model generates only few equilibria and thus allows to derive sharp predictions on behavior and welfare.

Finally, we discuss the need to look for mixed equilibria. While it is natural to focus on pure strategy equilibria when they exist, Proposition 5 shows that a pure strategy equilibrium does not always exist. For example, when $d, e < 0$, no pure equilibrium exists if $d > -(a+e)$

[29] For example, with $d, e < 0$ and $d > -(a+e)$, $d < -\frac{3}{5}a - \frac{1}{5}e$, this is the unique equilibrium when 1 is the first formateur (who selects 3 instead of the efficient 2); with $d, e > 0$ and $d > \frac{3}{7}a - \frac{5}{7}e$, $d < \min\{(a-e), \frac{3}{4} + \frac{e}{4}\}$, this is the unique equilibrium when 3 is the first formateur (who selects 2 instead of the efficient 1).

[30] For example, for $d, e \geq 0$, there is an efficient equilibrium for any order of formateurs if $d \leq \frac{3}{7}a - \frac{5}{7}e$ when $d, e \geq 0$, and $d \geq -\frac{3}{5}a - \frac{1}{5}e$ and $d \leq 3a + 7e$ if $d, e < 0$ the core is empty, but there is an efficient equilibrium in which 1 is the first proposer.

[31] As first proven by Eraslan [2002], in Baron and Ferejohn [1989] we typically have multiple stationary equilibria, but they all lead to the same equilibrium payoffs.

and $d < -\frac{3}{2}a - 2e$. This is a situation in which, if we believe in a literally infinite horizon, the equilibrium can be searched among the mixed equilibria; but if we believe the model has a finite but perhaps long horizon, then assuming the exact number of stages in the bargaining protocol is important. For completeness, in the online appendix, we have characterized as an example a mixed equilibrium in this region as well. In this equilibrium, party 1 randomizes between forming a coalition with 2 and 3; party 2 forms a coalition with party 1 with probability one, and party 3 forms a coalition with party 2 with probability one (see Figure 2 for an illustration).

5 Positive analysis

In this section we discuss how the model can help explain a number of empirical facts that have traditionally been seen as at odds with formal theories of legislative bargaining: the lack of a formateur's premium, supermajorities, and the presence of delays in forming the coalition.

The formateur's premium. A key prediction of standard noncooperative models a' la Baron and Ferejohn [1989] is that the party selecting the coalition receives a very significant premium in terms of surplus allocation. This reflects the fact that in these models the proposer has, at least temporary, monopoly power on the choice of the allocation and can exploit this advantage. A surprising but robust finding in the empirical literature is that not only does such a premium not exist, but that indeed formateurs may suffer a proposer's penalty.[32] For instance, Warwick and Druckman [2001] show that the payoff formateurs receive falls short of their vote contributions to the coalition by 13.3%.[33]

Proposition 5 makes clear that a positive formateur's premium is not a necessary prediction. For example, assume that the different coalitions generate similar valuations, so there is not an obviously superior or inferior coalition and we always have a counterclockwise equilibrium. Consider party 1 when it is the formateur: can it extract a formateur's premium? It is easy to see that this is not possible and indeed it will be very willing to concede a bonus to the coalition partner. If negotiation fails, 2 becomes proposer: 1 expects 2 to form a coalition with 3, leaving himself marginalized. This makes it rational for 1 to leave 2 more than 50% of the surplus generated in their coalition. Naturally, 1 can try with 3, but 3 would require an even higher surplus since he expects to be in a coalition with 2 in which, indeed, 2 will be willing to leave him more than 50% of the surplus (again, this because 2 fears that if proposal power movers to 3, then

[32] This literaure, started by Browne and Franklin [1973], typically relies on ministerial portfolio allocations (often weighted by the importance of the cabinets) as a measure of surplus allocation. See, among many others, Warwick and Druckman [2001] and [2006].

[33] This finding does not only rely on the fact that coalitions split surplus equally but the formaterur is systematically the largest party. Warwick and Druckman [2001] find that the coefficient of a variable interacting the size of a political party with a dummy equal to one when the party is formateur has a significantly negative sign.

3 will form a coalition with 1).[34] This is a manifestation of the *hold-up problem* in multilateral bargaining. Formateur's 1 can threaten 2 to switch to 3 while bargaining, but the threat would not be credible. Party 2 knows that 1 will either return to the table or fail as formateur. Party 2 then can comfortably hold 1 up and extract more than 50% of the surplus.[35]

Another manifestation of the hold up problem that explains a low (or indeed zero) formateur's premium will be presented in Section 6.2, where we discuss a version of our model in which the formateurs are randomly selected as in the original version of Baron and Ferejohn [1989]. In the benchmark case with symmetric players and symmetric coalitions, Baron and Ferejohn [1989] predicts that the formateur captures 2/3 of the surplus in the unique symmetric equilibrium in a coalition with 2 parties when $n = 3$; with our model with intracoalitional bargaining the formateur captures only 1/2 of the surplus in the unique symmetric equilibrium, so the formateur's premium is zero.

The characterization of Proposition 5, however, suggests a more important point regarding how surplus is distributed in government formation. Most of the discussion has focused on how surplus is divided between parties in the *realized* coalition. Failure of the formateur to capture more than his/her share has been interpreted as evidence that the formateur does not benefit from its proposal power. This is natural in a world in which all coalitions have the same value (as in standard non cooperative models) and in which the "grand coalition" is the most valuable coalition (as implicitly assumed in all cooperative models, that have little to say on the choice of coalition). In a model in which coalitions have heterogeneous values and equilibria may be inefficient, the formateur's benefit of proposal power mostly comes from the *choice of the coalition*, rather than from the *share of surplus* that is obtained. For example, party 1 obtains less than 50% of *a*-*d* when proposer in a counterclockwise equilibrium leading to a coalition $\{1, 2\}$, but even less than this if he attempts to form the more efficient coalition $\{1, 3\}$, and exactly zero if he loses proposal power (since $\{2, 3\}$ forms in this case). The real benefit of being formateur for party 1 is in selecting $\{1, 2\}$.

Supermajorities. In Section 4, we assumed that there is no intrinsic benefit in the size of a coalition. If η^* is the most efficient policy that can be achieved by some minimal winning coalition, then considering a supermajority can only add veto players, thus reducing the surplus

[34] The argument is indeed general and true for any party who may be proposer and indeed for any order of proposer.

[35] Another natural environment in which the formateur's premium does not need to occur is the case in which there is a clearly superior (and a clearly inferior) coalition ($d + e > a$). In this case too party 1 can not credibly extract a positive proposer's bonus. Party 3 knows that party 1 really has no choice, since the coalition $\{1, 2\}$ is so inferior. Parties 1 and 3 are basically stuck with each other, so 1 can expect to receive anything in $[a - d, e]$; and 3 can expect to receive the reminder $a + e - x_1$. Positive, zero or negative proposer's bonuses are now possible in equilibrium. Again, the hold-up problem is at work here: this time one in which 1 and 2 can hold each other up.

to be divided.[36] Formally, we assumed $V(\{1,2,3\}) \leq \max_C V(C)$. Under this assumption, a supermajority is never optimal in equilibrium. It is intuitive that the model presented above may explain the emergence of supermajorities if we drop this assumption and allow supermajorities to be more valuable than smaller coalitions. In a supermajority the formateur needs to compromise with more parties, but if the surplus is sufficiently large this may be worthwhile.

Consider the model with 3 parties studied above, but now assume that $V(\{1,2,3\}$ is strictly larger than $\max_{C\in\mathcal{C}\setminus\{1,2,3\}} V(C)$. By Proposition 3, the supermajority $\{1,2,3\}$ is chosen by all formateurs in equilibrium as $\Delta \to 0$ if its average payoff $V(\{1,2,3\}/3$ is larger than the average payoff of all the minimal winning coalitions, $V(C)/2$ with $n(C) = 2$. Proposition 3 also implies that when the supermajority is the coalition of all parties then, indeed for any number of parties, it is a strongly efficient equilibrium only if it has the largest average surplus.

In general, however, we might have a supermajority form even if the supermajority is not the coalition with the largest average surplus, and even if the core is empty. To see these phenomena it is useful have more than 3 parties. The 3 parties case is special when studying supermajorities since in this case the only feasible supermajority is the coalition of all players. When $n > 3$ instead we can have a supermajority and parties that are excluded from it, which seems the most natural case. As a simple example, consider an environment with 4 parties, $N = \{1,2,3,4\}$ in which the possible coalitions are $\{1,2\}$, $\{2,3\}$, $\{1,2,3\}$ or $\{1,3,4\}$ (see Figure 4 for an illustration). This situation can emerge with simple majority if 2 is, say, a large center right party (say the Christian Democrats) with 30% of the votes; 1 and 3 are two center left parties (say the Liberals and the Greens) with 25% of the votes each; and 4 is a smaller leftist party (say the Socialists) with 20% of the votes. Party 2 needs only one of the center left parties (and cannot form a government with the leftist party); the center left parties alone are not sufficient to form a center left government, they need the leftist party. So we may have either a center right minimal winning coalition (i.e. $\{1,2\}$ or $\{2,3\}$); a center right supermajority ($\{1,2,3\}$); or a center left supermajority ($\{1,3,4\}$). Assume that the set of formateurs is $N = \{1,2,3,4\}$, the order is $1 \to 2 \to 3 \to 4$, and the values are $V(\{1,3,4\}) = A$ and $V(\{1,2\}) = V(\{2,3\}) = V(\{1,2,3\}) = B$ with $A > B$.

In this scenario, it is unlikely that the efficient coalition $\{1,3,4\}$ emerges as a strongly efficient equilibrium: as $\Delta \to 0$, this would require $\{1,3,4\}$ to be three times as efficient as $\{1,2\}$ or $\{2,3\}$. However, $\{1,3,4\}$ still emerges in equilibrium under much less demanding conditions. Consider an equilibrium in which 1 forms a coalition $\{1,3,4\}$, 2 forms the coalition $\{1,2\}$, 3 forms the coalition

[36] In our formalization we have not specified policies, but they are implicit in the value function $V(C)$. The government chooses a policy η_C in coalition C that generates a surplus $s_i(\eta_C)$ to party $i \in C$. We can link η_C to C by letting $V(C) = \sum s_i(\eta_C)$. Assuming transferable utilities as in Austen-Smith and Banks [1988] and Baron and Diermeier [2001], each party can now achieve $u_i(\eta_C) = s_i(\eta_C) + t_i$, where naturally we need $\sum t_i \leq \sum s_i(\eta_C)$ and $t_i \in \left[-s_i(\eta_C), \sum_{l\neq i} s_l(\eta_C)\right]$.

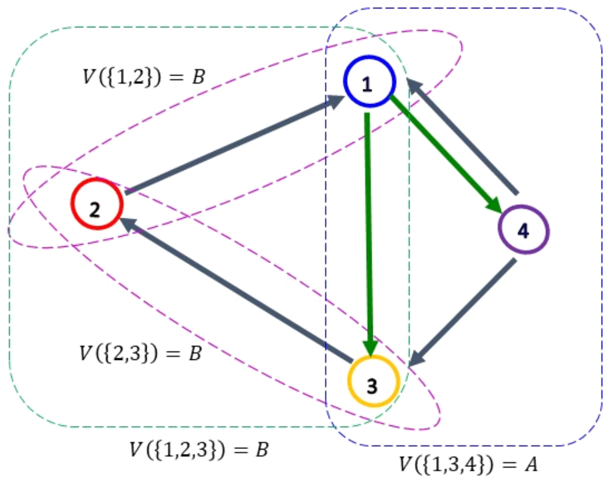

Figure 4: Supermajorities with an empty core: a four party example. The dashed line illustrate the feasible winning coalitions given the parties' sizes. The arrows illustrate the coalitions formed in and our of equilibrium if 1 fails. This equilibrium, exists if $A \in [B, (7/2)B]$.

$\{2,3\}$, and 4 forms $\{1,3,4\}$. If this equilibrium exists, as $\Delta \to 0$ the payoffs must satisfy:

$$
\begin{aligned}
x_1^1 &= x_1^2 + \frac{1}{3}\left(A - x_1^2\right), \, x_2^1 = 0, \, x_3^1 = \frac{1}{3}\left(A - x_1^2\right), \, x_4^1 = \frac{1}{3}\left(A - x_1^2\right) \\
x_1^2 &= \frac{1}{2}(B - x_2^3), \, x_2^2 = x_2^3 + \frac{1}{2}(B - x_2^3), \, x_3^2 = 0, \, x_4^2 = 0 \\
x_1^3 &= 0, \, x_2^3 = \frac{1}{2}(B - x_3^4), \, x_3^3 = x_3^4 + \frac{1}{2}(B - x_3^4), \, x_4^3 = 0 \\
x_1^4 &= x_1^1, \, x_2^4 = x_2^1, \, x_3^4 = x_3^1, \, x_4^4 = x_4^1.
\end{aligned}
$$

Which give us equilibrium payoffs $x_1^1 = \frac{5}{13}A + \frac{2}{13}B$, $x_2^1 = 0$, $x_3^1 = x_4^1 = \frac{4}{13}A - \frac{1}{13}B$ when the proposer is 1; $x_1^2 = \frac{1}{13}A + \frac{3}{13}B$, $x_2^2 = \frac{10}{13}B - \frac{1}{13}A$, $x_3^2 = x_4^2 = 0$ when the proposer is 2; $x_2^3 = \frac{7}{13}B - \frac{2}{13}A$, $x_3^3 = \frac{2}{13}A + \frac{6}{13}B$, $x_1^3 = x_4^3 = 0$ when the proposer is 3; and $x_1^4 = \frac{5}{13}A + \frac{2}{13}B$, $x_2^4 = 0$, $x_3^4 = x_4^4 = \frac{4}{13}A - \frac{1}{13}B$ when the proposer is 4. It can be verified that this is a limit equilibrium as $\Delta \to 0$ if $A \in \left[B, \frac{7}{2}B\right]$, so even if the coalition with 3 parties excluding 2 is only marginally more efficient than the other coalitions with two parties.

To see why this can be an equilibrium, note that 3 and 4 have incentives to take 1's offer since, if they do not, then they know that 2, in case of a bargaining breakdown when 1 is the formateur, will exclude them from the coalition. At the same time, 1 is not tempted to deviate and form a coalition with 2 because, in this case, it will have a strategic disadvantage vis-a-vis party 2: party 2 knows that in case of a breakdown, it will be formateur and it will have a larger outside option (since party 3 excludes party 1).

Note that having a party who is excluded from the supermajority (party 2) plays an important role in supporting the supermajority $\{1, 3, 4\}$. If the supermajority remained constant as formateurs change (because all formateurs propose $\{1, 3, 4\}$, or because some formateur fails to form a coalition and all others propose $\{1, 3, 4\}$), then the payoff would be split uniformly among the members of the supermajority as in Proposition 3: but then, the allocation would have to satisfy the same condition as in Proposition 3 and therefore be in the core (which is impossible if $\{1, 3, 4\}$ is only marginally better than the minimal winning coalitions). In the equilibrium constructed above, the division of surplus in the supermajority $\{1, 3, 4\}$ that induces party 1 to propose it and for 3 and 4 to accept it is supported by the expectation that the next formateur, party 2, forms a coalition $\{1, 2\}$ that includes 2 and excludes 3 and 4; in turn, this coalition is supported by the expectation that party 3 will again include 2 and exclude 1 and 4. The fact that the members of the equilibrium coalition shift as we change the formateur after a bargaining breakdown is critical in forming these expectations and therefore in defining the reservation utilities that justify the allocation of surplus in the supermajority.

Strategic delays. Proposition 3 shows that we may have strategic delay in equilibrium when a strongly efficient equilibrium exists as in Examples 1 and 2.[37] In the three party model of Section 4, strategic delays occur with a non empty core (i.e. if $a < |d| + |e|$) when, with $d, e > 0$, party 2 is unable to form a government and $\{1, 3\}$ is the only coalition that can be formed (if $e > a$); and when, with $d, e < 0$, 3 is unable to form a government and $\{1, 2\}$ is the only coalition that can be formed (if $|d| > a$). An interesting observation is that delays in our model are not due to the fact that there is an expected opportunity in the future that, by assumption, is too large not to be waited for (in terms of expected surplus). In the case with $d, e > 0$, for example, when $e \in \left(a, \frac{3}{2}a - \frac{d}{2}\right)$, *both* an efficient equilibrium with delay *and* an inefficient clockwise equilibrium coexist: party 2 is unable to form a government because of the players' (self-fulfilling) equilibrium beliefs. The delays in reaching an agreement, therefore, is a strategic phenomenon.

In the cases with delay, a formateur who is unable to form a government may decide to spontaneously give up his/her right to form a government, a possibility that is not contemplated by the model. Apart from the fact that the process of appointing the formateur and giving up the appointment takes time, there are many reasons why a party should want to receive the investiture as formateur and "give it a try" even if the probability of success is small or zero: s/he may enjoy the spotlight; s/he may see it as an opportunity to shift the responsibility of a delay

[37] Following the literature, equilibria with delays here are defined as equilibria in which at least a formateur is unable or unwilling to form a government: so in which government formation need to be paused until a new formateur is appointed. Naturally, in the limit case in which Δ is literally zero, failure of forming a government may involve no time delay. The results presented above, however, describe limits of sequences of equilibria with $\Delta > 0$, so with actual time delay. We focus on the limit case as $\Delta \to 0$ only for convenience.

on the other parties; s/he may hope in a change in the environment. Historically, this seems to be the case. For example, after the Italian election of 2018, the first appointed formateur was Elisabetta Casellati from *Forza Italia*: although her attempt was widely considered pro forma,[38] it took a week before a new formateur could be appointed. The appointment of Casellati as first formateur was considered a necessary step since she was the sitting president of the Senate.

Our model is not the first to show the possibility of strategic delays in multilateral bargaining. Variants of the basic model in which equilibria may have delays have been presented in environments in which, for example, the size of the pie to be divided changes stochastically during the negotiations; or in environments without common priors in which players may be optimistic about their recognition probabilities.[39] The results presented above contribute to this literature showing that strategic delays can arise in a model that does not have any of these features, and that can also explain negative formateur benefits and supermajorities.

6 Extensions and variations

6.1 Take-it-or-leave-it offers and intra-coalitional bargaining

An interesting special case of the model presented in Section 2 is the case in which the formateur makes a take-it-or leave-it offer (TIOLI) to the coalition partners and, if the offer is not accepted by the coalition, the formateur is replaced with probability one, as in Baron and Ferejohn [1989]. This case can be seen as a special case of our model in which p converges to 1.

When we have exogenous reservation utilities as in Section 3, Proposition 1 tells us that the formateur selects the coalition C_f^* that maximizes $V(C) - \sum_{i \in C} u_i$; and s/he appropriates all the surplus net of the reservation utilities of the coalition partners. When instead, as in Section 4, we endogenize the reservation utilities, as $p \to 1$ the extended model becomes a version of Baron and Ferejohn [1989] with a deterministic rotating order of proposers/formateurs.[40] The analysis in this case can be done as in Section 4. Consider the coalition formation process that we have in a "clockwise equilibrium" under the assumption that $d, e > 0$: that is, an equilibrium in which there is immediate agreement and 1 forms a coalition $\{1, 3\}$, 2 forms $\{1, 2\}$, and 3 forms $\{2, 3\}$.

[38] The secretary of the largest party, the 5 Star Movement, publicly excluded the possibility of accepting Casellati's mediation the very day of her appointment. See for instance *Repubblica* [2018].

[39] Previous stories about strategic delays extended the basic model to identify important economic factors that may cause them. Avery and Zemsky [1994], Diermeier et al. [2003], Acharia and Ortner [2013] and Ortner [2017] consider models in which the size of the pie to divide changes stochastically during the negotiations. Ali [2006] considers environments without common priors in which players may be optimistic about their recognition probabilities. Jehiel and Molduvanu [1995] and Iaryczower and Oliveros [2019] consider principal agent models in which the principal negotiates bilaterally with each agent.

[40] A version of the model in which the formateurs are randomly selected as in Baron and Ferejohn [1989] is presented in Section 6.2.

In this scenario, party 1 makes a TIOLI equal to x_3^2 to 3 and takes $x_1^1 = a+e-\beta x_3^2$ for itself: the offer is x_3^2 since this is the payoff 3 would receive from 2 if there is a bargaining breakdown with 1. Similarly, party 2 makes a TIOLI equal to x_1^3 to 1; and party 3 makes a TIOLI equal to x_2^1 to 2. If such an equilibrium exists, the payoffs are uniquely characterized by the system:

$$\begin{aligned} x_1^1 &= a+e-\beta x_3^2,\ x_2^1 = 0,\ x_3^1 = \beta x_3^2 \\ x_1^2 &= \beta x_1^3,\ x_2^2 = a-d-\beta x_1^3,\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = \beta x_2^1,\ x_3^3 = a-\beta x_2^1 \end{aligned} \tag{12}$$

This system has a unique solution as $\Delta \to 0$ in which each formateur fully expropriates the coalition partner: $x_1^1 = a+e$, $x_2^2 = a-d$, $x_3^3 = a$. Moreover, given these payoffs, it can be verified that the clockwise coalition formation process is indeed an equilibrium for any feasible d, e.

In Proposition 6 below we show that the clockwise equilibrium is indeed the unique pure strategy equilibrium when the core is empty, i.e. $|a| > |e| + |d|$.[41] We have:

Proposition 6. *If the core is empty, the game with three parties and TIOLI offers has a unique limit equilibrium in pure strategies as $\Delta \to 0$: the clockwise equilibrium in which the formateur forms the government with the party that is not next in line as formateur and fully extracts all the surplus.*

Comparing this result with Proposition 5, we can learn a lessons on how efficiency in coalition formation depends on the negotiating environment and why it is important to model intra-coalitional bargaining with $p < 1$. With an empty core, in the unique limit equilibrium with TIOLI offers, the formateurs follow a simple strategy that allows them to fully extract all the surplus: they make an offer to the party with the weakest bargaining position (that is, the party who is not next in line to be appointed as formateur), regardless of its contribution to the surplus of the coalition. In this case, whether we have an efficient equilibrium or not, does not at all depend on the values of the coalitions, but only on the order of the formateurs: the equilibrium is efficient if 1 is the first formateur; but it is inefficient otherwise. In the case in which 3 is the formateur, for example, the inefficient coalition $\{2, 3\}$ is formed despite the fact that 3 could form a coalition with 1. This happens because 3 knows that 1 is next in line as formateur, thus in a strong bargaining position because of the the assumption of take-it-or-leave-it-offers. In this process, the holdup problem is completely absent since the formateur extracts all the surplus net

[41] When the core is not empty, we can also have a strongly efficient equilibrium. Assume for example that $d, e > 0$. In a strongly efficient equilibrium with TIOLI we must have that $\{1, 3\}$ is formed and $x_1^1 = a+e-\beta^2 x_3^3$ and $x_3^3 = a+e-\beta x_1^1$. A $\beta \to 1$, if the equilibrium exists, the equilibrium payoffs therefore are $x_1^1 = x_1^3 = \frac{2}{3}(a+e)$ and $x_3^1 = x_3^3 = \frac{1}{3}(a+e)$ and $x_2^j = 0$ for $j = 1, 2, 3$. It follows that a strongly efficient equilibrium exists if 3 is not tempted to form with 2 (i.e. $\frac{1}{3}(a+e) > a$, or $e > 2a$). It can be verified that if 3 is not tempted to form a coalition with 2, then 1 will not be tempted to form a coalition with 2 as well.

of the reservation utilities. What makes this description of the strategic interaction between the parties unsatisfactory is the fact that, when party 1 is the formateur, party 3 knows that its participation is necessary to form an efficient coalition; and, importantly, that if 1 fails, *both* itself and 1 are left with zero utility by 2. Still, party 1 is able to capture the entire surplus of the partnership. Indeed, party 1's formateur's premium is independent of its reservation value and the reservation value of its partner. In such a situation it is unrealistic to assume that 1 can commit to a TIOLI offer; it would instead be natural to allow 1 and 3 to negotiate some more beyond the TIOLI before proposal power passes to 2: it is in the interest of both of them. This is however made impossible by the assumption that party 1's offer is take-it-or-leave-it.

It is important to stress that there is not an a priori "correct" way to model the bargaining procedure: the cases as $p \to 1$ (TIOLI) and $p \to 0$ (intra-coalitional negotiations) capture two different strategic environments, both of which may be relevant in different contexts. This discussion however shows that the assumptions matter for positive and welfare results. If we believe that a formateur/proposer can credibly make take-it-or-leave-it offers, then assuming $p \to 1$ as in Baron and Ferejohn [1989] is the correct option. In this case, we have a model that predicts an extreme version of the formateur's premium in which the formateur receives all the surplus. If instead we believe that a formateur needs to engage in meaningful negotiations with the coalition partners, then we have to solve for the general model with $p < 1$ and the natural case to consider is with $p \to 0$, as we have done in Propositions 2, 3 and 5. As seen in the previous analysis, the model with intra-coalitional bargaining can explain why we do not observe extreme allocations of "surplus" favoring the formateur in government formation processes, even if the order of formateurs is predictable.

6.2 Random recognition of formateurs

As highlighted by Ali, Bernheim and Fan [2019], it is often the case that in legislative (or other forms of) bargaining a formateur at stage t can predict who will be appointed at $t+1$ as formateur in case of bargaining breakdown; or at least, but no less crucially, s/he can predict who will not be appointed. This motivates the choice made above of a deterministic order of formateurs. In this section, we discuss the equilibria when we assume that formateurs are randomly selected as in Baron and Ferejohn [1989].

Consider the three party model presented in Section 4 with $d, e \geq 0$, but now assume that in case of bargaining breakdown each party $i = 1, 2, 3$ has a probability equal to $1/3$ to be selected as formateur. In this case, as $p \to 1$, our bargaining model becomes the model in Baron and Ferejohn [1989], where it is assumed that offers are TIOLI and formateurs are randomly selected; as $p \to 0$, we instead have a model of intracoalitional bargaining and random selection of formateurs.

Let us first compare intracoalitional bargaining to bargaining with TIOLI, focusing on the same "divide the dollar" environment assumed in Baron and Ferejohn's original paper (i.e. with $d = e = 0$). For both cases, consider a symmetric and anonymous equilibrium in which a formateur selects the partner among the other two parties with probability 1/2, which is the most popular workhorse in applications using multilateral bargaining with symmetric environments. In both models, the ex ante payoff of the players is $a/3$, the division of the surplus is however very different. Baron and Ferejohn's model has a unique symmetric equilibrium in which the formateur receives $x_f^{TIOLI} = a - \frac{\beta}{3}a$ and the coalition partner $x_{-f}^{TIOLI} = \frac{\beta}{3}a$. In the intracoalitional bargaining model the payoffs instead are:

$$\begin{aligned}
x_f^I &= \frac{\beta p}{1-\beta(1-p)} \cdot \frac{a}{3} + \frac{1-\beta(1-p)}{1-[\beta(1-p)]^2}\left[\frac{3(1-\beta(1-p))-2\beta p}{1-\beta(1-p)} \cdot \frac{a}{3}\right] \\
x_{-f}^I &= \frac{\beta p}{1-\beta(1-p)} \cdot \frac{a}{3} + \frac{\beta(1-p)[1-\beta(1-p)]}{1-[\beta(1-p)]^2}\left[\frac{3(1-\beta(1-p))-2\beta p}{1-\beta(1-p)} \cdot \frac{a}{3}\right]
\end{aligned}$$

As $\Delta \to 0$, in Baron and Ferejohn [1989] payoffs converge to $x_f^{TIOLI} = \frac{2}{3}a$ and $x_{-f}^{TIOLI} = \frac{1}{3}a$, so the formateur obtains 50% more than the coalition partner, despite having exactly the same outside option. This is not quite as large as the formateur's advantage we found when assuming TIOLI and a deterministic order of formateurs as in Section 6.1, but it is still a very large advantage.[42]

With intracoalitional bargaining, payoffs instead converge to $x_f^I = \frac{1}{2}a$ and $x_{-f}^I = \frac{1}{2}a$ as $\Delta \to 0$, so the formateur's advantage is zero. This seems a more sensible outcome in a completely symmetric environment in which the parties are ex ante identical in terms of their potential to generate surplus, they have the same recognition probabilities and they expect to be treated symmetrically.

When we generalize the environment assuming heterogeneous coalitional values (i.e., $d, e > 0$) with random recognitions, *both* the analyses with TIOLI and with intracoalitional bargaining become significantly more complicated because, as can be easily verified, in both protocols pure strategy clockwise and counterclockwise equilibria no longer exist. Moreover, since with heterogeneous coalitions the model is no longer symmetric, we can't rely on symmetric strategies to simplify the analysis. In both models, as we change d, e, it is not just the mixing probabilities, but also the identity of the parties who mix that changes, making the characterization a complex function of the parameters.

To complete the analysis, we have computed as an example the "quasi-symmetric" mixed equilibria with TIOLI and intracoalitional bargaining in an environment in which all coalitions are the same except that one coalition is better than the others: so $V(\{1,2\} = V(\{2,3\} = a$ and

[42] Indeed, it is this smaller but still very large advantage in the protocol with random formateurs that critics of noncooperative models of legislative bargaining have in mind when they lament excessive and empirically unsupported theoretical predictions for the formateur's premium (see for example Laver [1998]).

$V(\{1,3\} = a + e$. This case is (relatively) easier to analyze because party 1 and 3 are equivalent in the eyes of party 2, so an (anonymous) mixed equilibrium involves 2 selecting 1 and 3 with probability 1/2 each; and 1 and 3 selecting party 2 with probability $1-\alpha$. It can be verified that, as $\Delta \to 0$, this equilibrium exists in both protocols when $e \leq 3/(5a)$, and α is equal to $\alpha^{TIOLI} = \left[4e - 3a + \sqrt{28e^2 - 12ae + 9a^2}\right] / (4e)$ with TIOLI, and to $\alpha^{I} = (3a + 3e) / (6a - 2e)$ with intracoalitional bargaining. Both values converge to 1/2 as $e \to 0$, but the payoffs converge to, respectively, $x_f^{TIOLI} = \frac{2}{3}a$ and $x_{-f}^{TIOLI} = \frac{1}{3}a$ and to $x_f^{I} = \frac{1}{2}a$ and $x_{-f}^{I} = \frac{1}{2}a$. Again, the second solution seems more plausible in an environment with highly patient players, who have (in the limit) the same potential for surplus and same outside options.

We should finally note that while, in a model with symmetric payoffs, the assumption that formateurs are selected with uniform random probabilities with replacement makes sense, it becomes much less natural when coalitions have heterogeneous values and so parties have different potentials to generate surplus. In a symmetric environment, the assumption of uniform random recognitions gives us a "super-symmetric" and "super-stationary" model that is a natural benchmark to capture stylized features of negotiations. With heterogeneous coalitions, however, this type of "super-symmetry" and "super-stationarity" is impossible, so it is inevitable to take a stand on the order of formateurs or, if the order is random, on the probabilities of recognition. We leave to future research the characterization of the equilibria of the model under alternative assumptions on the recognition probabilities in its two extreme cases as $p \to 0$ and $p \to 1$.

6.3 Random recognition of proposers

In the model of Sections 2-4, we have assumed that in the intracoalitional bargaining stage, after C has been selected, the order of proposers is deterministic, following some order $\iota(\tau, C)$. An alternative possibility is that the proposer at stage τ is randomly selected. The analysis presented above can be extended without complications to such environments for a variety of natural random recognition rules. Assume in this section for simplicity and without loss of generality that $\beta = 1$ and consider, for example, the following rule: after the formateur's first proposal at $\tau = 1$, the proposer in the following stages is randomly selected with uniform probability $1/(n(C) - (\tau - 1))$ among the $n(C) - (\tau - 1)$ parties in C who have not yet proposed. In this case, the formateur's payoff at $\tau = 1$ can be written as:

$$
\begin{aligned}
x^*_{f,f}(C, C_f) &= V(C) - p \sum_{j \in C \setminus f} u_j \\
&\quad - (1-p) \sum_{k \in C \setminus \{f\}} \left[\left(\frac{1}{n(C)-1} \right) \left(\begin{array}{c} \sum_{j \in C \setminus \{f,k\}} a^{(2)}_{k,j}(C, C_f) \\ + V(C) - \sum_{j \in C \setminus \{k\}} a^{(2)}_{k,j}(C, C_f) \end{array} \right) \right] \\
&= p \left[V(C) - \sum_{j \in C \setminus f} u_j \right] + (1-p) \left(\frac{1}{n(C)-1} \right) \sum_{k \in C \setminus \{f\}} a^{(2)}_{k,f}(C, C_f)
\end{aligned}
\tag{13}
$$

where $a^{(2)}_{k,j}(C, C_f)$ is the acceptance threshold of j when k is the proposer at the second round of proposals. As in the preceding analysis, these acceptance thresholds can be shown to be independent of j and functions of only u_f and $x^*_{f,f}(C_f, C_f)$. In the online appendix, we indeed show that, with the recognition rule we have just described, they can be written as:

$$
a^{(2)}_{j,f}(C, C_f) = p \sum\nolimits_{k=0}^{n(C)-2} (1-p)^k u_f + (1-p)^{n(C)-1} x^*_{f,f}(C_f, C_f) \tag{14}
$$

which, when inserted in (13), bring us back to (5) and allows us to characterize the equilibrium.

The random recognition rule does not need to have the feature that all parties in C are recognized as proposers before the formateur is recognized again. Two natural conditions need to be satisfied to allow a representation in which $x^*_{f,f}(C, C_f)$ depends only on the formateur's reservation utility u_f and the net total surplus $V(C) - \sum_{l \in C} u_l$. First, we need that the first proposal comes from the formateur, which is quite natural since it is the formateur who selects the coalition. Second, we need that the first counterproposal comes form a party different than the formateur, which also seems quite natural given that is a "counterproposal."

To see this point, consider the case in which offers come from the formateur in odd stages, and from a party in $j \in C \setminus \{f\}$ in even stages, where j is randomly selected with probability q_j, with $q_j \geq 0$ for $j \in C \setminus \{f\}$ and $\sum_{j \in C \setminus \{f\}} q_j = 1$. The formateur's payoff can be written as:

$$
x^*_{f,f}(C, C_f) = p \left[V(C) - \sum_{j \in C \setminus f} u_j \right] + (1-p) \sum_{k \in C \setminus \{f\}} q_j \cdot a^{(2)}_{k,f}(C, C_f) \tag{15}
$$

In this case, the formateur's acceptance threshold for any proposer $j \in C \setminus \{f\}$ is given by:

$$
a^{(2)}_{j,f}(C, C_f) = p u_f + (1-p) x^*_{f,f}(C_f, C_f) \tag{16}
$$

since after the randomly selected party at stage 2, proposal power returns to the formateur who turns to coalition C_f. Substituting $a^{(2)}_f(C, C_f)$ from (16) in (15) we obtain a condition analogous to (5) in which $x^*_{f,f}(C, C_f)$ can be solved as the solution of a simple equation and is a function of u_f and $V(C) - \sum_{l \in C \setminus f} u_l$ alone. Once we have $x^*_{f,f}(C, C_f)$ for all C we can solve for C_f and the equilibrium payoffs of the other players as we have done in the analysis above.

We can assume even more complex and realistic processes and obtain qualitatively similar results. For example, we could assume that the probability that the formateur is selected to propose is gradually increasing in τ: so that it is zero at round 2, where the other parties are allowed to "respond" to the first proposal; but then it becomes increasingly significant (perhaps equal to one by stage $n(C)+1$), so that the formateur has an increasing probability to recalibrate his/her offer, or to change coalition. As we change the recognition rule, and thus the bargaining advantage of the formateur in the intracoalitional bargaining, we naturally change the formateur's share of surplus and we can thus calibrate the model to alternative institutional settings.

6.4 Discounting and the order of limits

In the previous section we have maintained the assumption that, as $\Delta \to 0$, the dominant concerns for the parties is the probability of a bargaining breakdown, that is $(1 - \beta_\Delta)/p_\Delta \to 0$. As anticipated, however, the analysis carries through in a qualitatively similar way if we assume $(1 - \beta_\Delta)/p_\Delta \to \theta$ for some finite θ. Consider for example the counterclockwise equilibrium with $n = 3$ characterized in (8)-(9). The equilibrium conditions become:[43]

$$x_1^1 = \left(a - d - \xi x_2^2\right)/2,\ x_2^1 = \left(a - d + \xi x_2^2\right)/2, x_3^1 = 0 \tag{17}$$

$$x_1^2 = 0,\ x_2^2 = \left(a - \xi x_3^3\right)/2,\ x_3^2 = \left(a + \xi x_3^3\right)/2$$

$$x_1^3 = \left(a + e + \xi x_1^1\right)/2,\ x_2^3 = 0,\ x_3^3 = \left(a + e - \xi x_1^1\right)/2,$$

where $\xi = 1/(1+\theta)$. System (17) has a unique solution, continuous in θ, converging to the solution of (8)-(9) as $\theta \to 0$. The equilibrium payoffs are now a function of θ and similarly the conditions for the existence of this equilibrium depend on θ, but they can be derived following the same approach as in the previous sections.

6.5 Externalities

In the previous analysis we considered an abstract environment in which the coalition forming the government, say C, generates a surplus $V(C)$ that is shared among its members, leaving the others at zero. This is a straightforward extension of the environment in Baron and Ferejohn

[43] In terms of the general characterization of Proposition 1, we have that $\beta p/\left(1 - \beta\left(1 - p\right)\right) \to 1/(1 + \theta)$ and $[\beta(1-p)]^{\iota^{-1}(i,C)-1}\left[1 - \beta(1-p)\right]/\left[1 - [\beta(1-p)]^{n(C)}\right] \to 1/n(C)$ when $(1 - \beta)/p \to \theta$ as $\Delta \to 0$.

[1989], where the winning coalition shares a pie of size one among its members, leaving the others at zero. If we think that the surplus generated by C depends on some policy η, it is natural to allow it to have externalities on the other parties as well. In this section we first show that, in our environment with transferable utilities, this more complex scenario is in line with the model presented above. In the online appendix we show that the bargaining model easily extends to the case with externalities even if we assume imperfectly transferable utilities (or not transferable at all). Without loss of generality, we assume here for simplicity that $\beta = 1$.

Consider an environment in which party $i \in N$ has utility $U_i(\eta, z_i) = u_i(\eta) + z_i$, where η is a policy from some space P, z_i is a transfer and $u_i(\cdot)$ is a concave function. For example, the policy space P could be a subspace of R^m for some integer m (the dimensionality of the policy) and $u_i(\eta)$ could be the usual quadratic distance $u_i(\eta) = \vartheta + \gamma \sum_l (\eta_l - \eta_l^i)^2$ with $\vartheta > 0$ and $\gamma < 0$, $\eta^i = \left(\eta_l^i\right)_{l=1}^m$ is i's ideal point and $\eta = (\eta_l)_{l=1}^m$ is a policy in P.[44] To each party i, we associate a subset $P_i \subseteq X$ of policies that are acceptable to its constituency (for example, abortion is not a policy feasible for Republicans in the U.S.; and a flat tax is not a policy feasible to Democrats). Let $P_C = \cap_{j \in C} P_j$. This is the set of policies feasible to a coalition C.

A coalition C can select any policy in P_C and a vector of transfers/taxes z_j for $j = 1, ..., n$. A policy η now does not only affect the members of C, but all parties in N. Note, however, that a winning coalition, i.e. a legitimate government, can tax or subsidize all parties, including those outside the coalition.[45] We assume that no party can be left with less then its reservation utility that we set at zero. This level reflects the fact that there are checks on governmental power. Formally, we assume that $z_j \geq -u_j(\eta)$ and $\sum z_j \leq 0$, the latter inequality being a budget balance condition.

Given this, we have that any coalition C sets $z_j = -u_j(\eta)$ for all $j \notin C$ and allocates a surplus $V(C) = \max_{\eta \in P_C} \sum_{j \in N} u_j(\eta)$ to all members of C. As in the model described in Section 2, all parties outside the coalition are left with zero. The allocation in the coalition will be a vector of utilities x_j such that $x_j \geq 0$, $\sum_j x_j \leq V(C)$, exactly as the set $X(C)$ defined in Section 2. In this context, when the outside options u_i are exogenous simply corresponds to a policy η^o and feasible vector or transfers that would be taken by a caretaker: $u_i = u_i(\eta^o) + z_i^o$. When the outside options are endogenous, they are endogenously defined in the game.

[44] For a model with preferences like this see, for example, Baron and Diermeier [2001].

[45] The parties in our game represent different social classes, they are not individuals. It is therefore legitimate to allow the government to target them differentially with tax/subsidies.

7 Conclusions

In this paper we have proposed a new model of multilateral bargaining to study how majorities are formed in legislatures when coalitions are heterogeneous in terms of the surplus they are expected to generate. In our model, a formateur picks a coalition and negotiates for the allocation of the surplus. The formateur is free to change coalition to seek better deals with other coalitions, but s/he may lose her status if bargaining breaks down, in which case a new formateur is chosen. In this context, a formateur needs to reconcile the need to form the most productive coalition with the desire to maximize the share of output that s/he captures. This seems an important feature that has characterized most legislative negotiations in parliamentary democracies in the post World War II period.

The model provides a new perspective on legislative bargaining and helps explain a number of well established empirical facts at odds with existing noncooperative models of multilateral bargaining. From a theoretical point of view, we have shown that, as the delay between offers goes to zero, the equilibrium allocation converges to a generalized version of the Nash bargaining solution in which each coalition member receives its outside option plus a share of surplus net of reservation utilities. The difference with respect to the Nash's solution is that in the n-person Nash bargaining solution the coalition is assumed to be comprised by all players (or chosen exogenously), while in our model it is *endogenously* determined. A form of the hold-up problem specific to these bargaining games may contribute to generate significant inefficiencies in the selection of the equilibrium coalition. When reservation utilities are endogeneized in a fully recursive model in which a bargaining breakdown is followed by the appointment of a new formateur, moreover, we may have multiple stationary equilibria with different welfare implications. The equilibrium characterization is however sufficiently tight for positive and normative analysis.

In terms of positive analysis, the model helps explaining three well known empirical facts that have been hard to reconcile with non-cooperative models of multilateral bargaining: the absence of significant (or even positive) premia in ministerial allocations for formateurs and their parties; the occurrence of supermajorities; and delays in reaching agreements. While a number of important previous works have attempted to explain these facts individually, our theory has the advantage of providing a unified and intuitive explanation for all them.

8 Appendix

8.1 Proof of Lemma 1

Let $x^*_{f,f}(C, C_f)$ be the formateur's payoff when C is proposed, but the equilibrium coalition is C_f. We must have that $x^*_{f,f}(C, C_f) = V(C) - \sum_{i \in C \backslash f} a_{f,i}(C, C_f)$ where, as we did in Section 3, we are using the notation $a_{j,i}(C, C_f)$ to indicate the acceptance threshold of i when j proposes in coalition C and the expected equilibrium coalition is C_f to emphasize that the threshold depends on C_f. It follows that:

$$\begin{aligned} x^*_{f,f}(C, C_f) &= V(C) - \beta p \sum_{l \in C \backslash f} u_l - \beta(1-p) \left[\begin{array}{c} \sum_{l \in C \backslash \{f, \iota(2,C)\}} a_{\iota(2,C),l}(C, C_f) \\ +V(C) - \sum_{k \in C \backslash \{\iota(2,C)\}} a_{\iota(2,C),k}(C, C_f) \end{array} \right] \\ &= [1 - \beta(1-p)] \, V(C) - \beta p \sum_{l \in C \backslash f} u_l + \beta(1-p) \left[a_{\iota(2,C),f}(C, C_f) \right] \qquad (18) \\ &= [1 - \beta(1-p)] \left[V(C) - \frac{\beta p}{1 - \beta(1-p)} \sum_{l \in C \backslash f} u_l \right] + \beta(1-p) \left[a_{\iota(2,C),f}(C, C_f) \right] \end{aligned}$$

Note now that we must have: $a_{\iota(2,C),f}(C, C_f) = \beta p u_f + \beta(1-p) a_{\iota(3,C),f}(C, C_f)$. Iterating this formula $n(C) - 2$ times we have:

$$a_{\iota(2,C),f}(C, C_f) = p \sum_{k=0}^{n(C)-2} (1-p)^k \beta^{k+1} u_f + [\beta(1-p)]^{n(C)-1} \, x^*_{f,f}(C_f, C_f)$$

Substituting this expression in (18), we conclude that in equilibrium we must have:

$$x^*_{f,f}(C_f, C_f) = \max_{C \in \mathcal{C}_f} \left\{ \begin{array}{c} [1 - \beta(1-p)] \left[V(C) - \frac{\beta p}{1 - \beta(1-p)} \sum_{l \in C} u_l \right] \\ +\beta p \left[1 + \sum_{k=1}^{n(C)-1} (1-p)^k \beta^k \right] \cdot u_f + [\beta(1-p)]^{n(C)} \, x^*_{f,f}(C_f, C_f) \end{array} \right\}. \qquad (19)$$

Recalling that C_f is a coalition that solves (19), from (19) we immediately have that:

$$\begin{aligned} x^*_{f,f}(C_f, C_f) &= \frac{\beta p \cdot \sum_{k=0}^{n(C_f)-1} (1-p)^k \beta^k}{1 - [\beta(1-p)]^{n(C_f)}} u_f + \frac{1 - \beta(1-p)}{1 - [\beta(1-p)]^{n(C_f)}} \left[\begin{array}{c} V(C_f) \\ -\frac{\beta p}{1 - \beta(1-p)} \sum_{l \in C_f} u_l \end{array} \right] \\ &= \frac{\beta p}{1 - \beta(1-p)} u_f + \frac{1 - \beta(1-p)}{1 - [\beta(1-p)]^{n(C_f)}} \left[V(C_f) - \frac{\beta p}{1 - \beta(1-p)} \sum_{l \in C_f} u_l \right] \qquad (20) \end{aligned}$$

Assume now that we have an equilibrium in which a C_f that is not a solution of (4) is chosen.

Then can write:

$$
\begin{aligned}
x^*_{f,f}(C^*_f, C_f) &= [1-\beta(1-p)]\left[V(C^*_f) - \frac{\beta p}{1-\beta(1-p)}\sum_{l\in C^*_f} u_l\right] \\
&\quad +\beta p\left[1+\sum_{k=1}^{n(C^*_f)-1}(1-p)^k\beta^k\right]\cdot u_f + [\beta(1-p)]^{n(C^*_f)}\, x^*_{f,f}(C_f, C_f) \\
&= x^*_{f,f}(C_f, C_f) + \left[1-[\beta(1-p)]^{n(C^*_f)}\right]\left[\begin{array}{c} \frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C^*_f)}}\left[V(C^*_f) - \frac{\beta p}{1-\beta(1-p)}\sum_{l\in C^*_f} u_l\right] \\ -\frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C)}}\left[V(C_f) - \frac{\beta p}{1-\beta(1-p)}\sum_{l\in C_f} u_l\right] \end{array}\right] \\
&> x^*_{f,f}(C_f, C_f).
\end{aligned}
$$

Implying that indeed C_f does not solve the problem in (19), a contradiction. Similarly we have that $x^*_{f,f}(C_f, C^*_f) \leq x^*_{f,f}(C^*_f, C^*_f)$ for any $C_f \in \mathcal{C}^f$: we conclude that the unique fixed-point of (3) is C^*_f. ■

8.2 Proof of Proposition 1

From Lemma 1 we know that one and only one coalition is chosen by the formateur, C^*_f that is the unique fixed-point of (3). We now show that there is a unique distribution of surplus and we characterize it. Let $a^*_{\iota\left(\tau+j,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f)$ be the acceptance threshold of the party who proposes at stage τ when the proposer is the party proposing at stage $\tau+j$. Moreover, $x^*_{\iota\left(\tau+j,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f)$ is the payoff of $\iota\left(\tau, C^*_f\right)$ when $\iota\left(\tau+j, C^*_f\right)$ is the proposer.

Note that we must have: $a^*_{\iota\left(\tau+j,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f) = \beta p u_{\iota\left(\tau,C^*_f\right)} + \beta(1-p) a_{\iota\left(\tau+j+1,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f)$. Iterating, we can then write:

$$
a^*_{\iota\left(\tau+j,C^*_f\right),\iota\left(\tau,C^*_f\right)} = \beta p \sum_{k=0}^{n(C^*_f)-j-1}[\beta(1-p)]^k\, u_{\iota\left(\tau,C^*_f\right)} + [\beta(1-p)]^{n(C^*_f)-j}\, x^*_{\iota\left(\tau,C^*_f\right),\iota\left(\tau,C^*_f\right)} \tag{21}
$$

for all $j = 1, ..., n(C^*_f) - \tau$. Similarly we have:

$$
a^*_{\iota\left(\tau-j,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f) = \beta p \sum_{k=0}^{j-1}[\beta(1-p)]^k\, u_{\iota\left(\tau,C^*_f\right)} + [\beta(1-p)]^j\, x^*_{\iota\left(\tau,C^*_f\right),\iota\left(\tau,C^*_f\right)}(C^*_f) \tag{22}
$$

for all $j = 1, ..., \tau - 1$. Moreover, following the same steps as in the derivation of (20), we have:

$$
x^*_{\iota\left(\tau,C^*_f\right),\iota\left(\tau,C^*_f\right)} = \frac{\beta p}{1-\beta(1-p)} u_{\iota\left(\tau,C^*_f\right)} + \frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C^*_f)}}\left[\begin{array}{c} V(C^*_f) \\ -\frac{\beta p}{1-\beta(1-p)}\sum_{l\in C^*_f} u_l \end{array}\right] \tag{23}
$$

The system of equations (23), (21) and (22) gives a complete characterization of the optimal strategy for the agent proposing at stage τ in C_f^* (that is party $\iota(\tau, C_f^*)$). Clearly, we must have $x^*_{\iota(\tau+j,C_f^*),\iota(\tau,C_f^*)}(C_f^*) = a^*_{\iota(\tau+j,C_f^*),\iota(\tau,C_f^*)}(C_f^*)$. The strategies and equilibrium payoffs are fully characterized by the system of $n(C_f^*) \times n(C_f^*)$ equations:

$$\begin{aligned}
x^*_{\iota(\tau,C_f^*),\iota(\tau,C_f^*)} &= \frac{\beta p}{1-\beta(1-p)} u_{\iota(\tau,C_f^*)} + \frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C_f^*)}} \begin{bmatrix} V(C_f^*) \\ -\frac{\beta p}{1-\beta(1-p)} \sum_{l \in C_f^*} u_l \end{bmatrix} \\
x^*_{\iota(\tau+j,C_f^*),\iota(\tau,C_f^*)} &= \begin{bmatrix} \beta p \sum_{k=0}^{n(C_f^*)-j-1} (1-p)^k \beta^k u_{\iota(\tau,C_f^*)} \\ + [\beta(1-p)]^{n(C_f^*)-j} \, x^*_{\iota(\tau,C_f^*),\iota(\tau,C_f^*)} \end{bmatrix} \text{ for } j = 1, ..., n(C_f^*) - \tau \qquad (24) \\
x^*_{\iota(\tau-j,C_f^*),\iota(\tau,C_f^*)} &= \beta p \sum_{k=0}^{j-1} (1-p)^k \beta^k u_{\iota(\tau,C_f^*)} + [\beta(1-p)]^j \, x^*_{\iota(\tau,C_f^*),\iota(\tau,C_f^*)} \text{ for } j = 1, ..., \tau - 1
\end{aligned}$$

for all $\tau \in \{1, ..., n(C_f^*)\}$. It is immediate to verify that C_f^* and the strategies described in (24) are an equilibrium. To characterize the equilibrium payoffs for the players note that:

$$\begin{aligned}
x^*_{f,\iota(\tau,C_f^*)} &= a^*_{f,\iota(\tau,C_f^*)} = \beta p \sum_{k=0}^{\tau-2} [\beta(1-p)]^k \, u_{\iota(\tau,C_f^*)} + [\beta(1-p)]^{\tau-1} \, x_{\iota(\tau,C_f^*),\iota(\tau,C_f^*)}(C_f^*) \qquad (25) \\
&= \frac{\beta p}{1-\beta(1-p)} u_{\iota(\tau,C_f^*)} + \frac{[\beta(1-p)]^{\tau-1}\,[1-\beta(1-p)]}{1-[\beta(1-p)]^{n(C_f^*)}} \begin{bmatrix} V(C_f^*) \\ -\frac{\beta p}{1-\beta(1-p)} \sum_{l \in C_f^*} u_l \end{bmatrix}
\end{aligned}$$

for all $\tau \geq 2$. ■

8.3 Proof of Proposition 2

From Proposition 1, the limit of the formateur's payoff as $\Delta \to 0$ can be written as:

$$\lim_{\Delta \to 0} x^*_{f,f}(\Delta) = \lim_{\Delta \to 0} \left[\frac{\beta_\Delta p_\Delta}{1-\beta_\Delta(1-p_\Delta)} u_f + \frac{1-\beta_\Delta(1-p_\Delta)}{1-[\beta_\Delta(1-p_\Delta)]^{n(C_f^*)}} \left[V(C_f^*) - \frac{\beta_\Delta p_\Delta}{1-\beta_\Delta(1-p_\Delta)} \sum_{l \in C_f^*} u_l \right] \right].$$

Applying *l'Hospital* rule, we have:

$$\lim_{\Delta \to 0} x^*_{f,f}(\Delta) = \frac{1}{1+\lim_{\Delta \to 0} |\beta'_\Delta / p'_\Delta|} u_f + \frac{1}{n(C_f^*)} \left[V(C_f^*) - \frac{1}{1+\lim_{\Delta \to 0} |\beta'_\Delta / p'_\Delta|} \sum_{l \in C_f^*} u_l \right]$$

By Assumption 1, we have: $\lim_{\Delta \to 0} |\beta'_\Delta / p'_\Delta| = \lim_{\Delta \to 0} (1-\beta_\Delta) \backslash p_\Delta = 0$. Applying Assumption 1, we therefore have:

$$\lim_{\Delta \to 0} x^*_{f,f}(\Delta) = u_f + \frac{1}{n(C_f^*)} \left[V(C_f^*) - \sum_{l \in C_f^*} u_l \right]$$

Moreover, from (24) and (25) that:

$$\lim_{\Delta\to 0}\left(\frac{x_i^*(\Delta)-\frac{\beta_\Delta p_\Delta}{1-\beta_\Delta(1-p_\Delta)}u_i}{x_f^*(\Delta)-\frac{\beta_\Delta p_\Delta}{1-\beta_\Delta(1-p_\Delta)}u_f}\right)=\lim_{\Delta\to 0}\left[[\beta_\Delta(1-p_\Delta)]^{\iota^{-1}(i,C_f^*)-1}\right]=1.$$

It follows that $\lim_{\Delta\to 0}\left(x_f^*(\Delta)-\frac{\beta_\Delta p_\Delta}{1-\beta_\Delta(1-p_\Delta)}u_f\right)=\lim_{\Delta\to 0}\left(x_i^*(\Delta)-\frac{\beta_\Delta p_\Delta}{1-\beta_\Delta(1-p_\Delta)}u_i\right)$, proving the result. ■

8.4 Proof of Proposition 3

We proceed in three steps. In Step 3.1, we characterize the payoffs in a strongly efficient equilibrium. In Step 3.2, we show that $(n(C\cap C^{**})\backslash n(C^{**}))\cdot V(C^{**})\geq V(C)$ is necessary for existence of a strongly efficient equilibrium as $\Delta\to 0$. Finally, in Step 3.3, we show that $(n(C\cap C^{**})\backslash n(C^{**}))\cdot V(C^{**})>V(C)$ is sufficient as $\Delta\to 0$. Let $x_j^i(\Delta)$ be the payoff of j when the formateur is i for a given Δ. Moreover, for all $i\in N$, let $\gamma_i(l)$ be lth formateur in C^{**} that is in the formateur line after i.

Step 3.1. Assume we are in a strongly efficient equilibrium in which coalition C^{**} is selected by all formateurs who can form a coalition. Consider the payoff of a party $j\in C^{**}$ with formateur $i\in C^{**}\cap\mathcal{T}$ when, after a bargaining breakdown, the formateurs next in line are k followed by l with $k\notin C^{**}$. The reservation utility of j is:

$$\begin{aligned}u_j^i(\Delta) &= x_j^k(\Delta)=\beta\left[p\cdot x_j^l(\Delta)+(1-p)x_j^k(\Delta)\right]\\ &\Leftrightarrow\ u_j^i(\Delta)=\frac{\beta p}{1-\beta(1-p)}\cdot x_j^l(\Delta)\end{aligned}$$

Iterating this logic, we have that for any $j\in C^{**}$:

$$u_j^i(\Delta)=\left[\frac{\beta p}{1-\beta(1-p)}\right]^{L_i-1}\cdot x_j^{\gamma_i(1)}(\Delta)$$

where L_i is the number of formateur following i until formateur $\gamma_i(1)\in C^{**}\cap\mathcal{T}$ is appointed (so the number of formateurs not in C^{**} between i and $\gamma_i(1)$ plus one). From (6), we therefore have:

$$\begin{aligned}x_i^i(\Delta) &= \left(\frac{\beta p}{1-\beta(1-p)}\right)\cdot u_i^i(\Delta)+\frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C^{**})}}\left[\begin{array}{c}V(C^{**})\\ -\left(\frac{\beta p}{1-\beta(1-p)}\right)\sum_{l\in C^{**}}u_l^i(\Delta)\end{array}\right] \qquad (26)\\ &= \left(\frac{\beta p}{1-\beta(1-p)}\right)^{L_i}\cdot x_i^{\gamma_i(1)}(\Delta)+\frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C^{**})}}\left[\begin{array}{c}V(C^{**})\\ -\left(\frac{\beta p}{1-\beta(1-p)}\right)^{L_i}\sum_{l\in C^{**}}x_l^{\gamma_i(1)}(\Delta)\end{array}\right]\\ &= \xi^{L_i}\cdot x_i^{\gamma_i(1)}(\Delta)+\frac{1-\beta(1-p)}{1-[\beta(1-p)]^{n(C^{**})}}\left[1-\xi^{L_i}\right]V(C^{**})\end{aligned}$$

where in the last line we use the fact that in a strongly efficient equilibrium $\sum_{l\in C^{**}} x_l^{\gamma_i(1)}(\Delta) = V(C^{**})$ and we define $\xi = \beta p \backslash (1-\beta(1-p))$.

Note that, following similar steps as in the derivation of $x_i^i(\Delta)$, we have:

$$x_i^{\gamma_i(1)}(\Delta) = \xi^{L_{\gamma_i(1)}} \cdot x_i^{\gamma_i(2)}(\Delta) + \frac{[\beta(1-p)]^{\iota^{-1}(i,C^{**},\gamma_i(1))-1}[1-\beta(1-p)]}{1-[\beta(1-p)]^{n(C^{**})}}\left[1-\xi^{L_{\gamma_i(1)}}\right]V(C^{**}) \tag{27}$$

where $\iota(\tau, C_f^*, j)$ is the identity of the proposer at stage τ when the coalition is C_f^* and the formateur is j; and $\iota^{-1}(i, C_f^*, j)$ is the position in the formateurs' line of i when the coalition is C_f^* and the formateur is j. Combining (26) and (27) and iterating, we obtain:

$$\begin{aligned} x_i^i(\Delta) &= \prod\nolimits_{l\in C^{**}\cap\mathcal{T}} \xi^{L_l} \cdot x_i^i(\Delta) + \sum_{j=0}^{n(C^{**}\cap\mathcal{T})-1}\left[\begin{array}{c} \prod_{k=0}^{j}\xi^{L_{\gamma_i(k-1)}}\left[1-\xi^{L_{\gamma_i(j)}}\right] \\ \cdot\frac{[\beta(1-p)]^{\iota^{-1}(i,C^{**},\gamma_i(j))-1}[1-\beta(1-p)]}{1-[\beta(1-p)]^{n(C^{**})}}V(C^{**}) \end{array}\right] \\ \Leftrightarrow \quad x_i^i(\Delta) &= \frac{1}{1-\prod_{l\in C^{**}\cap\mathcal{T}}\xi^{L_l}} \sum_{j=0}^{n(C^{**}\cap\mathcal{T})-1}\left[\begin{array}{c} \prod_{k=0}^{j}\xi^{L_{\gamma_i(k-1)}}\left[1-\xi^{L_{\gamma_i(j)}}\right] \\ \cdot\frac{[\beta(1-p)]^{\iota^{-1}(i,C^{**},\gamma_i(j))-1}[1-\beta(1-p)]}{1-[\beta(1-p)]^{n(C^{**})}}V(C^{**}) \end{array}\right] \end{aligned}$$

where we note that $\gamma_i(0) = i$ and we define $L_{\gamma_i(-1)} = 0$ and $\iota^{-1}(i, C^{**}, \gamma_i(0)) = 1$. From (6) we also have:

$$x_j^i(\Delta) = \frac{1}{1-\prod_{l\in C^{**}\cap\mathcal{T}}\xi^{L_l}} \sum_{m=0}^{n(C^{**}\cap\mathcal{T})-1}\left[\begin{array}{c} \prod_{k=0}^{m}\xi^{L_{\gamma_i(k-1)}}\left[1-\xi^{L_{\gamma_i(m)}}\right] \\ \cdot\frac{[\beta(1-p)]^{\iota^{-1}(j,C^{**},\gamma_i(m))-1}[1-\beta(1-p)]}{1-[\beta(1-p)]^{n(C^{**})}}V(C^{**}) \end{array}\right]$$

for all $j \in C^{**}\backslash i$ and $i \in C^{**}\cap\mathcal{T}$. Taking the limits we have $x_j^i(\Delta) \to V(C^{**})/n(C^{**})$ for all $j \in C^{**}$ and $i \in C^{**}\cap\mathcal{T}$.

Step 3.2. We now prove that $(n(C\cap C^{**})\backslash n(C^{**}))\cdot V(C^{**}) \geq V(C)$ is necessary for a strongly efficient equilibrium to exist. Assume by contradiction that $(n(C\cap C^{**})\backslash n(C^{**}))\cdot V(C^{**}) < V(C)$ for some coalition C. There must be a $\zeta > 0$ such that, for any sequence Δ_m converging to zero as $m\to\infty$, there is a coalition C_m such that $\xi_m \frac{n(C_m\cap C^{**})}{n(C^{**})}\cdot V(C^{**}) < V(C_m) - \zeta$ for m sufficiently large (where $\xi_m = \beta(\Delta_m)p(\Delta_m)\backslash[1-\beta(\Delta_m)(1-p(\Delta_m))]$, so $\xi_m \to 1$ as $m\to\infty$). Let $L^- = \min_k L_k$. From Step 3.1, for any arbitrarily small ε, there is a m^* such that for $m > m^*$ and any $l \in C^{**}$:

$$(\xi_m)^{L^-}\sum\nolimits_{j\in C_m\cap C^{**}} x_j^l(\Delta_m) < (\xi_m)^{L^-}\frac{n(C_m\cap C^{**})}{n(C^{**})}\cdot V(C^{**}) + \varepsilon(\xi_m)^{L^-} < V(C_m) - \epsilon$$

for $\epsilon = \zeta - (\xi_m)^{L^-}\varepsilon > 0$, since ε can be chosen arbitrarily small and $\zeta > 0$. Assume that there is a formateur $k \in C_m \cap (N \backslash C^{**})$. For $m > m^*$, k can form a coalition C_m and obtain a payoff:

$$\begin{aligned} x'_k &= \xi_m \cdot u^k_k(\Delta_m) + \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}}\left[V(C_m) - \xi_m \cdot \sum\nolimits_{j\in C_m} u^k_j(\Delta_m)\right] \\ &\geq \xi^{L_k}_n \cdot x^{k'}_k(\Delta_m) + \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}}\left[V(C_m) - \xi^{L_k}_m \cdot \sum\nolimits_{j\in C_m\cap C^{**}} x^{k'}_j(\Delta_m)\right] \\ &\geq \xi^{L_k}_m \cdot x^{k'}_k(\Delta_m) + \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}} \cdot \epsilon > \xi^{L_k}_m \cdot x^{k'}_k(\Delta_m) = 0 \end{aligned}$$

where k' is the first formateur following k who is in C^{**} and the first inequality follows from the fact that $x^{k'}_j(\Delta_m) = 0$ for $j \notin C_m \cap C^{**}$. Since the payoff that k receives if s/he does not form a government is $\xi^{L_k}_m \cdot x^{k'}_k(\Delta_m) = 0$, we have a contradiction. Assume now that there is no formateur $k \in C_m \cap (N \backslash C^{**})$. Since the coalition must have a proposer, it must that there is a formateur $k \in C_m \cap C^{**}$. Note that there is a $\varsigma > 0$ such that $\frac{n(C_m\cap C^{**})}{n(C^{**})} \cdot V(C^{**}) < V(C_m) - \varsigma$; and there is a $m^{**} \geq m^*$ such that, for $m > m^{**}$, $(1-\xi^{L_k}_m)\frac{n(C^{**})-n(C_m\cap C^{**})}{n(C^{**})}V(C^{**}) < \frac{\varsigma}{4}$ and $\sum_{j\in C^{**}\cap C_m} x^{k'}_j(\Delta_m) < \frac{n(C_m\cap C^{**})}{n(C^{**})}V(C^{**}) + \frac{\varsigma}{2}$, where again k' is the first formateur in C^{**} following k. It follows that for $m > m^{**}$, by proposing C_m, k obtains:

$$\begin{aligned} x'_k &\geq \xi^{L_k}_m \cdot x^{k'}_k(\Delta_m) + \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}}\left[V(C_m) - \xi^{L_k}_m \cdot \sum\nolimits_{j\in C_m\cap C^{**}} x^{k'}_j(\Delta_m)\right] \\ &> \xi^{L_k}_m \cdot x^{k'}_k(\Delta_m) + \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}}\left[(1-\xi^{L_k}_m)V(C^{**}) + \nu\right] > x^k_k(\Delta_m) \end{aligned}$$

where $\nu = \left(1 - \frac{\xi^{L_k}_m}{2}\right)\varsigma - (1-\xi^{L_k}_m)\frac{n(C^{**})-n(C_m\cap C^{**})}{n(C^{**})}V(C^{**}) > \varsigma/4$. This is a contradiction.

Step 3.3. Finally, we show that if $(n(C \cap C^{**}) \backslash n(C^{**}))\cdot V(C^{**}) > V(C)$ then we have a strongly efficient equilibrium. Assume by contradiction that it is not true that we have a strongly efficient equilibrium on a sequence $\Delta_m \to 0$ as $m \to \infty$. Then for any m sufficiently large there is a formateur $i_m \in \mathcal{T}$ who finds it optimal to deviate from C^{**} and form a coalition C_m by offering payoffs $x'_k(\Delta_m)$ for $k \in C_m$. We must have a $\zeta > 0$, such that $[\xi_m]^{L^+}\frac{n(C_m\cap C^{**})}{n(C^{**})} \cdot V(C^{**}) > V(C_m) + \zeta$ for m sufficiently large, where $L^+ = \max_k L_k$. Moreover, for any arbitrarily small ε, there is a m^{***} such that for $m > m^{***}$:

$$[\xi_m]^{L^+}\sum\nolimits_{j\in C_m\cap C^{**}} x^{i'_m}_j(\Delta_m) \geq [\xi_m]^{L^+}\frac{n(C_m\cap C^{**})}{n(C^{**})} \cdot V(C^{**}) - [\xi_m]^{L^+}\varepsilon > V(C_m) + \epsilon$$

for $\epsilon = \zeta - [\xi_m]^{L^+}\varepsilon > 0$, where i'_m is the first formateur in C^{**} following i_m. It follows that $\xi_m \sum_{j\in C_m\cap C^{**}} u^{i_m}_j(\Delta_m) > V(C_m)$, so if $i_m \in C_m \cap (N \backslash C^{**})$, s/he would be unable to form the coalition C_m, a contradiction. If instead $i_m \in C_m \cap C^{**}$, then we would have

$$\begin{aligned} x'_{i_m}(\Delta_m) &\leq \xi^{L_{i_m}}_m \cdot x^{i'_m}_{i_m}(\Delta_m) + \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}}\left[V(C_m) - \xi^{L_{i_m}}_m \cdot \sum\nolimits_{j\in C_m\cap C^{**}} x^{i'_m}_j(\Delta_m)\right] \\ &\leq \xi^{L_{i_m}}_m \cdot x^{i'_m}_{i_m}(\Delta_m) - \frac{1-\beta_m(1-p_m)}{1-[\beta_m(1-p_m)]^{n(C_m)}}\epsilon < x^{i_m}_{i_m}(\Delta_m) \end{aligned}$$

Again, a contradiction. ■

8.5 Proof of Proposition 4

The proofs is presented in the online appendix. ■

8.6 Proof of Proposition 5

We proceed in two steps. Lemma A.5.1. characterizes the clockwise equilibrium.

Lemma A.5.1 *A clock-wise equilibrium exists as $\Delta \to 0$ only if $d \leq \min\left(\frac{3}{4}a + \frac{e}{4}, 3a - 2e\right)$ when $d, e > 0$ and if $d \geq -\frac{3}{2}a - 2e$ when $d, e \leq 0$; and it exists and is the limit of equilibria as $\Delta \to 0$ if these conditions are strict inequalities.*

Proof. We proceed in three steps. We first characterize the equilibrium payoffs assuming that a clockwise equilibrium exists; we then characterize when the strategies are optimal responses.

Step 1. Let x_j^i be the equilibrium surplus captured by j if i is the formateur. Starting with formateur 1, we must have:

$$x_1^1 = \xi x_1^2 + \phi\left(a + e - \xi x_1^2 - \xi x_3^2\right),\ x_2^1 = 0,\ x_3^1 = \xi x_3^2 + \phi\left(a + e - \xi x_1^2 - \xi x_3^2\right). \tag{28}$$

where $\xi = \beta p / [1 - \beta(1-p)]$ and $\phi = [1 - \beta(1-p)] / \left[1 - (\beta(1-p))^2\right]$. These formula follows from (6) using as outside options the equilibrium values received if the attempt of 1 fails, so formateur 2 is selected. Similarly as in (28) we have:

$$\begin{aligned} x_1^2 &= \xi x_1^3 + \phi\left(a - d - \xi x_1^3 - \xi x_2^3\right),\ x_2^2 = \xi x_2^3 + \phi\left(a - d - \xi x_1^3 - \xi x_2^3\right),\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = \xi x_2^1 + \phi\left(a - \xi x_2^1 - \xi x_3^1\right),\ x_3^3 = \xi x_3^1 + \phi\left(a - \xi x_2^1 - \xi x_3^1\right). \end{aligned}$$

Equations (28) and (29) give us a system of 9 equations in 9 unknowns, admitting a unique solution that, as $\Delta \to 0$, converges to:

$$\begin{aligned} x_1^1 &= \frac{6a + 5e - 2d}{9},\ x_2^1 = 0,\ x_3^1 = \frac{3a + 4e + 2d}{9} \\ x_1^2 &= \frac{3a + e - 4d}{9},\ x_2^2 = \frac{6a - e - 5d}{9},\ x_3^2 = 0 \\ x_1^3 &= 0,\ x_2^3 = \frac{3a - 2e - d}{9},\ x_3^3 = \frac{6a + 2e + d}{9}. \end{aligned} \tag{29}$$

A necessary condition for the clockwise strategies to be an equilibrium as $\Delta \to 0$ is that no party has a profitable deviation in the limit given (29). A sufficient condition for the clockwise strategies to be the limit of equilibria with $\Delta > 0$ as $\Delta \to 0$ is that they are a strict equilibrium in the limit.

Step 2. We now characterize the conditions under which no party has a profitable deviation in the limit when $d, e \geq 0$. When 1 is the formateur, by Proposition 2, we have that $\{1, 3\}$ is formed if it maximizes the average surplus of the coalition when 1 is the formateur. Let $S_1(C)$ be the average surplus in coalition C when 1 is the formateur. We have that: $S_1(\{1,3\}) = \left(a + e - x_1^2\right)/2 = (6a + 4d + 8e)/18 > 0$ and $S_1(\{1,2\}) = S_1(\{1,3\}) - \left(e + d + x_2^2\right)/2 \leq S_1(\{1,3\})$. Thus we have $S_1(\{1,3\}) > S_1(\{1,2\})$ and $S_1(\{1,3\}) > 0$, implying that $\{1,3\}$ is formed in equilibrium. When 2 is the formateur, $\{1,2\}$ is formed if $S_2(\{1,2\}) \geq S_2(\{2,3\})$. We have: $S_2(\{2,3\}) = \frac{1}{2}\left(a - \sum_{j=2,3} x_j^3\right) = 0$ and $S_2(\{1,2\}) = \frac{1}{2}\left(a - \sum_{j=1,2} x_j^3\right) = \frac{6a-8d+2e}{18}$. We therefore have $S_2(\{1,2\}) \geq S_2(\{2,3\})$ if and only if $d \leq \frac{3}{4}a + \frac{e}{4}$, with a strict inequality if $d < \frac{3}{4}a + \frac{e}{4}$. When 3 is the formateur, $\{2,3\}$ is formed if $S_3(\{2,3\}) \geq S_3(\{1,3\})$. We have: $S_3(\{2,3\}) = a - \sum_{j=2,3} x_j^1 = \frac{6a-4e-2d}{9}$ and $S_3(\{1,3\}) = a - \sum_{j=1,3} x_j^1 = 0$. It follows that the condition is satisfied if and only if $d \leq 3a - 2e$, and it is satisfied strictly if $d < 3a - 2e$. We conclude that when $d, e \geq 0$ a clockwise equilibrium exists only if $d \leq \min\left\{3a - 2e, \frac{3}{4}a + \frac{1}{4}e\right\}$, and it exists and is the limit of equilibria if this inequality is strict.

Step 3. We now characterize the conditions under which no party has a profitable deviation in the limit when $d, e < 0$. Consider first the case in which 1 is the formateur. We have: $S_1(\{1,3\}) = \left(a + e - x_1^2\right)/2 = (6a + 8e + 4d)/18$ and $S_1(\{1,2\}) = 0$. It follows that $S_1(\{1,3\}) \geq S_1(\{1,2\})$ if and only if $d \geq -\frac{3}{2}a - 2e$, and the inequality is strict if $d > -\frac{3}{2}a - 2e$. Consider now formateur 2. We have $S_2(\{1,2\}) = \frac{1}{2}\left(a - d - x_2^3\right) = (6a - 8d + 2e)/18$ and $S_2(\{2,3\}) = 0$, so $S_2(\{1,2\}) \geq S_2(\{2,3\})$ if $6a - 8d + 2e \geq 0$ and the first inequality is strict if the second is strict. Note that if the condition for formateur 1 is verified, i.e. $d \geq -\frac{3}{2}a - 2e$, then $2e \geq -\frac{3}{2}a - d$, so $6a - 8d + 2e \geq \frac{9}{2}a - 9d > 0$, implying that formateur 2 always finds it optimal to follow the strategies of the clockwise equilibrium. Finally consider formateur 3. We have: $S_3(\{2,3\}) = \left(a - x_2^1 - x_3^1\right)/2 = (6a - 4e - 2d)/18 > 0$ and $S_3(\{1,3\}) = 0$, so $S_3(\{2,3\}) \geq S_3(\{1,3\})$ is always true. We conclude that when $d, e < 0$ a clockwise equilibrium exists only if $d \geq -\frac{3}{2}a - 2e$ and it exists and is the limit of equilibria if $d > -\frac{3}{2}a - 2e$. ■

We now prove the existence of a counterclockwise equilibrium.

Lemma A.5.2. *A counterclockwise equilibrium exists as $\Delta \to 0$ only if $d \leq \frac{3}{7}a - \frac{5}{7}e$ when $d, e > 0$, and if $d \geq -\frac{3}{5}a - \frac{1}{5}e$ and $d \leq 3a + 7e$ when $d, e < 0$; and it exists and is the limit of equilibria as $\Delta \to 0$ if these inequalities are strict.*

Proof. The proof of this result is similar to the proof in Lemma A.5.1 and presented in the online appendix. ■

Lemmata A.5.1-A.5.2 define the thresholds presented in Proposition 5. ■

8.7 Proof of Proposition 6

The proofs of Proposition 6 is presented in the online appendix. ■

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